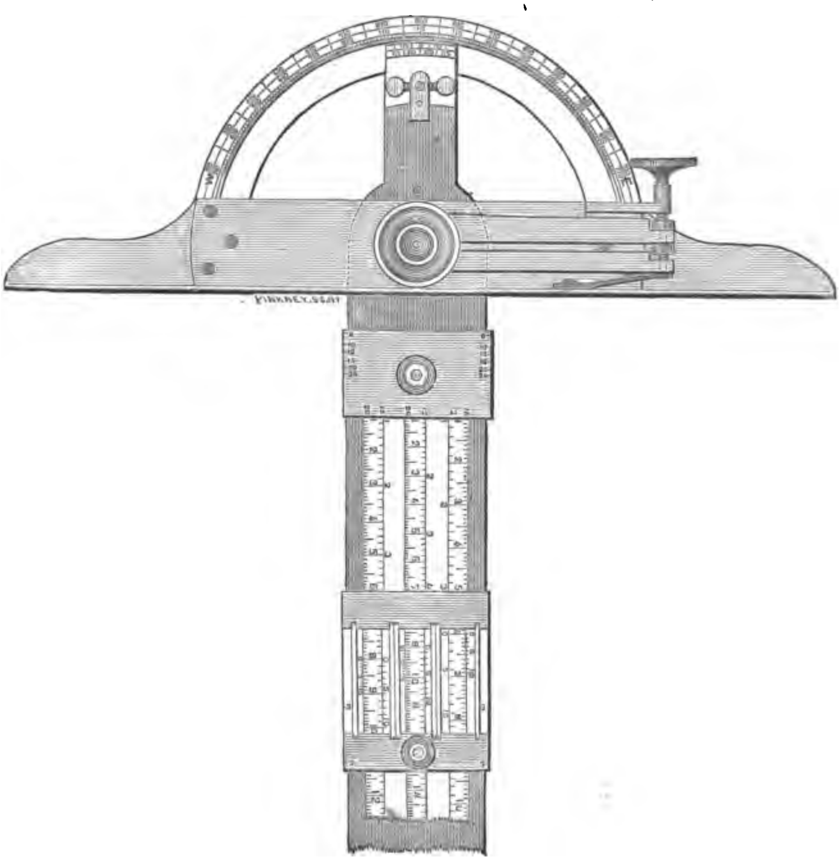
THE TRIGONOMETER.



A

MANUAL

OF THE

PROTRACTING TRIGONOMETER,

WITH ITS APPLICATION TO

RECTILINEAR DRAUGHTING AND PLOTTING,

TRIGONOMETRY, AND SURVEYING.

ILLUSTRATED BY EXAMPLES AND ENGRAVINGS.

BY JOSIAH LYMAN, A.M.,
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PREFACE.

FROM the time of Galileo to the present, perfected instruments for accurately measuring angles and distances, have been among the great desiderata of science. And by aid of the improvements furnished through the present achievements of art, the accuracy of those instruments with which the angles and distances are obtained, that supply the mathematician with the necessary data for his laborious investigations, is truly surprising.

A machine or instrument, by whose aid these data could be *mechanically reduced*, with an accuracy like that attained in the former class of instruments; or even one that would perform mechanically, and with only the accuracy of the common field instruments, all the *ordinary trigonometrical calculations* hitherto solved by the *Traverse Tables* and *Logarithms*, would be of inestimable utility to practical men.

In the recent invention of the *Protracting Trigonometer*, the result of several years' study and experiment by the author, the want seems to be fully met. It is the achievement of this instrument, that it so far eliminates, by its peculiar adjustments all the essential errors of manufacture, that it enables the operator to transfer to paper the data obtained by field or other instruments, and *deduce all the solutions of Plane Trigonometry to a minute of a degree in angle, and the thousandth of an inch in distance.*

Nor is it in mere mechanical execution that the excellence of this instrument consists. Its chief value lies in the great *saving of time and labor* it secures, both in *plotting outlines*, and in *trigonometrical calculations.* In the former it combines accuracy with expedition and ease; in the latter, it greatly simplifies and abbreviates the operations; rendering the solution of tedious mathematical problems an agreeable mechanical recreation. Hence

to any one who has become familiar with the use of the instrument, and its application to the questions of Mensuration, it appears not extravagant to say in general, that it saves *half the time and labor* usually employed in working them by Logarithms and the Traverse Table.

To describe the Trigonometer, to explain and illustrate its varied uses and applications is the object of this Manual.

The first section describes the parts, combination and general methods of using the instrument. This will be found sufficient for all the common purposes of rectilinear draughting; such as Architecture, Map-making, Plans for Machinery, Engraving, Patents, Railroad-sketches, &c., &c.

The second section shows the application of the instrument to the solution of the problems of Plane Trigonometry.

The remainder of the book is devoted to the detailed methods of operating the instrument in the Mensuration of Areas and the Division of Lands. No examples, however, are given, except what are necessary simply to illustrate the more obvious uses of the Trigonometer.

For additional examples more fully elucidating its varied applications, teachers, students, and surveyors are referred to *Stoddard* and *Henkle's* complete text-book on *Trigonometry and Land Surveying*, soon to be published by SHELDON & Co., 115 Nassau Street, New York.

THE AUTHOR.

LENOX, November, 1861.

TO PURCHASERS.

CLASSES AND PRICES OF THE TRIGONOMETER.

Plain Brass Instruments, 20 inch Rule and 15 inch Scale Plate,								$22.00
Brass Instruments with Tangent Fixture, same length of parts,								28.00
" " " "	22 inch rule & 16 inch scale plate,							30.00
" " " "	30	"	"	"	"	"		32.00
German Silver Instruments,	22	"	"	"	"	"		35.00
" " "	30	"	"	"	"	"		37.00

Extra lengths furnished to order.

For all trigonometrical operations, the instrument with 22 inch rule is the best. When it is required chiefly for drawing very large plans or maps, the 30 inch rule is preferable. Some times *both rules* are taken with the same instrument, and exchanged for each other at pleasure. This is done by merely taking out with a wide-edged screw driver the main pivot.

PRICES OF DRAUGHTING BOARDS.

Smaller size, 20 by 24 inches,	Iron border, ...	$3.50
" " " "	Steel "	4.00
Larger size 25 by 30 inches,	Iron "	6.00
" " " "	Steel "	6.50

Extra sizes made to order.

The prices given above are the retail prices, and considering the amount of labor required in the manufacture of the instruments, are very low, as every one will see who examines them. They are offered thus reasonably in order to secure for them a rapid introduction, and bring them within the reach of all.

Orders may be addressed to the manufacturer, L. H. Crane, Brattleboro', Vt., to the inventor, Josiah Lyman, Lenox, Mass., or to W. & L. E. Gurley, Troy, N. Y., his local Agents.

The instruments may be sent per Express directly to the purchaser; the

Express Agent at the place of delivery receiving the pay both for transportation, and the instrument.

A liberal discount is made when several are sent to one address.

The Manual, also a Microscope for the nicer operations are sent with the instrument to purchasers without charge.

W. & L. E. Gurley's Manual of Engineers' and Surveyors' Instruments, will be sent free to purchasers when desired.

Cases are sent with the instrument whenever ordered.

Of these the following are the prices for the 22 inch rule and 16 inch scale plate:

Plain Butternut, ...	$1.50
Mahogany, ...	2.00

The Trigonometers and Draughting Boards are adjusted and tested at the manufacturer's shop; and in all cases are warranted correct, and in good condition; the manufacturer and inventor agreeing in case of evident defect appearing after *suitable trial*, to repair or replace the same with a new and perfect instrument, promptly, and at their own cost, Express charges included.

RECOMMENDATIONS.

FROM EDMUND BLUNT, ESQ.,

Math. and Astron. Instrument Maker, and First Assistant in the U. S. Coast Survey.

"I have examined an instrument for protracting angles and distances on paper, such as are required by surveyors; it being, I believe, an original arrangement of MR. J. LYMAN, of Lenox, Mass.; and I consider it the most convenient Protractor in use. The sliding scale attached to it gives it an advantage over all other instruments of the kind."

FROM ALBERT HOPKINS,

Professor of Natural Philosophy and Astronomy in Williams' College.

"I was glad of the opportunity furnished by your call, to examine your new instrument, the *Trigonometer*, of which I had before heard. It strikes me as a decided advance upon any thing I have seen intended to answer the same purpose. You are able by its aid to lay down angles and lines with an exactness limited only by the nicety of the graduation, and of the adjustments, which latter seem to be quite under your control. The former of course must depend upon the artist, whom I should judge to be extremely competent, from the specimen I saw of his work.

Your instrument, in fact, enables the operator to introduce into the business of plotting, an accuracy like that which the astronomer attains in determining his data; certainly if you apply the microscope, which you might easily do.

In addition to the uses for which you primarily intended it, I think your instrument will be found of essential service in the projection of eclipses, and other delicate operations of a like kind."

FROM Z. RICHARDS,

Principal of Union Academy, Washington, D. C.

"Having had an opportunity to examine your *Trigonometer*, and to test its adaptation and accuracy, and also to compare it with the very best protracting instruments now used by the best surveyors connected with the Government of the United States, as well as with several other protracting

instruments of merit, I do not hesitate to give my decided preference to yours, having found it fully equal to all you claim for it.

For protracting, plotting, dividing and measuring the contents of land, easily, expeditiously and accurately, your instrument is unequaled, and should be in the hands of every teacher, and person, who has any thing to do with teaching, or with practicing the art of surveying."

From Henry S. Kelsey,

Surveyor and Instructor in Mathematics, in Amherst College.

"I have had the pleasure of examining an instrument called the *Protracting Trigonometer*, invented by Mr. Josiah Lyman, of Lenox, Mass.

With this instrument angles and lines may be laid down, with an accuracy far surpassing that attainable with any other instrument I have ever seen. It is so perfect in its work that areas may be obtained mechanically by it, with as much satisfaction as by the use of the Traverse Table, and I doubt not with one third or one quarter of the labor. For the practical surveyor this instrument will be invaluable; and I shall be greatly surprised if it does not come into very general use."

From E. S. Snell,

Professor of Mathematics and Natural Philosophy, in the same College.

"I have made a brief examination of Mr. Lyman's instrument, and feel disposed to concur fully in the foregoing statements of Mr. Kelsey."

From H. L. Eustis,

Professor of Civil Engineering, in the Lawrence Scientific School, Harvard University.

"I have examined an instrument patented by Mr. Lyman, of Lenox, Mass., and called by him the *Trigonometer*. As an instrument for plotting surveys and calculating areas it possesses great accuracy, and in the hands of a careful draughtsman would save a great deal of time and labor. It should be in the office of every surveyor."

From H. A. Newton,

Professor of Mathematics, in Yale College.

"The plotting instrument of Mr. Lyman, seems to me after a full examination of it, very valuable to surveyors, and one that will save a vast amount of labor in computing areas of fields and other tedious operations."

From W. A. Norton,

Professor of Civil Engineering, in Yale College.

"I have examined Mr. Josiah Lyman's *Protracting Trigonometer*, and am satisfied that it is admirably adapted to the purposes for which it is designed. It is compact and every way convenient for use; and appears to correspond in accuracy as a protracting instrument to the surveying instruments used in the field. Many of the determinations in which numerical calculations are now found necessary, can be effected graphically and much more expeditiously by the use of this new instrument, which combines the ordinary protractor, T square, and linear scale."

From Alex. C. Twining,

Practical Surveyor, and Professor of Mathematics and Civil Engineering, New Haven, Conn.

"The demand for accuracy in graphical processes and delineations is constantly increasing, so that instruments which ten years ago would have been considered mere beautiful exhibitions of skill and nicety, will now be esteemed of great practical value. On the other hand a *universal protractor* as I may term it, such as you have invented and patented, which is far more nice and accurate than any hitherto employed, will both satisfy the demand and augment it. Every man who has constant occasion for plotting and calculating areas or distances for ordinary purposes will, I think, find it an object to possess your *Protractor*."

An entire concurrence in the above statements has been expressed by Prof. J. Tatlock, of William's College; E. A. Hubbard, Surveyor and Teacher in Williston Seminary; Professors J. S. Benedict, and J. F. Stoddard, of New York; Prof. C. S. Lyman, of Yale College; J. H. French, Esq., Superintendent of the New York State Map; Geo. P. Bond, Esq., Observer at the Cambridge Observatory; Prof. O. M. Mitchell, Director of the Cincinnati Observatory; by Mathematical Instrument Makers, Delineators in the U. S. Coast Survey and Land Offices; as well as other Practical Surveyors, Architects, and distinguished teachers, in various sections of the country.

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THE TRIGONOMETER.

SECTION I.

DESCRIPTION OF THE PARTS, COMBINATION, AND GENERAL METHODS OF USING THE INSTRUMENT.

ARTICLE (1). DEFINITIONS.

1. **The Distance** of a line, is its rectilinear length.*

2. **A Meridian**† is a line that runs directly North or South, in the plane of the visible horizon, or on the map of a field.

3. **A Parallel of Latitude** is a line running directly East or West through any point, at right angles to the Meridian which passes through the same point.

4. **A Circle** is a plane figure, bounded by one line called the *circumference*, from which all straight lines drawn to a certain point within the figure are equal to each other; and this point is called the *center* of the circle. Thus ABE, Fig. 1., is a circle; of which F is the center, ABCDE the circumference; and each of the straight lines AF, BF, CF, DF, is called a *radius* of the circle.

5. **A Diameter** of a circle is a straight line passing through its center, and terminated both ways by the circumference; as

* When confined to *Land Surveying*, the distance of a line is its *horizontal* length.

† All Meridians passing through any survey of ordinary extent may be considered straight, parallel lines.

Strictly, Meridians are *vertical circles*, cutting the horizon at its North and South points at right angles. Hence no two of them are any where exactly parallel.

So also strictly, a parallel of latitude is a *circle* passing through the earth at right angles to its axis. It is called a *parallel*, because it is parallel to the plane of the Equator.

In ordinary field operations in which the compass is used, and also in mapping or plotting, the *direction of the needle*, for the sake of convenience, is for the time assumed as the meridian; the *true meridian* and the *variation of the needle* always being appended when the map is finished.

AC, Fig. 1. It divides the circle into two equal parts called *semicircles.*

FIG. 1.

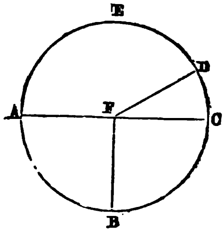
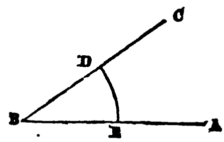


FIG. 2.



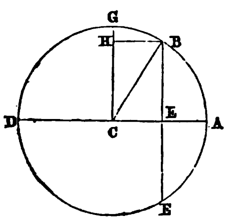
6. **An Arc** is any portion of the circumference; as BC, or CD, Fig. 1.

7. **A Quadrant** is the fourth part of a circle; as AFB, Fig. 1.

8. The circumference of every circle is supposed to be divided into 360 equal parts, called *degrees;* each degree into 60 equal parts, called *minutes;* and each minute into 60 equal parts, called *seconds,* &c.

9. **The Measure** of an angle is the arc intercepted between the two lines that form the angle; and the angular point is the center of the circle. Thus the angle ABC, Fig. 2, is measured by the arc DE, and contains the same number of degrees that the arc does.

FIG. 3.



Hence, the measure of a right angle is 90 degrees; for DG, Fig. 3, which measures the right angle DCG, is a fourth part of the circumference, or 90 degrees.

Degrees are usually indicated by a small circle, thus (°); minutes thus (′); and seconds thus (″); 20° 15′ 25″ therefore denotes that the arc or angle contains 20 degrees, 15 minutes, and 25 seconds.

10. **The Complement** of an arc or angle, is the difference between the arc or angle and 90°; and the *supplement* of an arc or angle, is what it wants of 180°. Thus BG, Fig. 3, is the complement of the arc AB; and BCG is the complement of the angle ACB.

So also the arc BD is the supplement of the arc AB; and BCD is the supplement of the angle ACB.

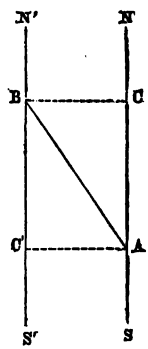
11. The sum of the three angles of a triangle is equal to two right angles, or 180°.

Hence, if the sum of any two angles of a triangle be subtracted from 180°, the remainder will be the third angle. And if one of the angles be subtracted from 180°, the remainder will be the sum of the other two angles.

Hence, also, if the triangle be right angled, and one of the acute angles be subtracted from 90°, the remainder will be the other acute angle.

12. **The Bearing** or **Course** of a line, is the angle which it makes with a meridian passing through one of its terminal points. It is reckoned from the North or South points of the horizon towards the East or West points. Thus, supposing NS, Fig. 4, a meridian, the angle NAB, is the bearing or course of the line AB ; and if it contains 35°, it is read North 35° West; or N. 35° W.

Fig. 4.



13. **The Reverse Bearing** of a line, is the bearing taken from the other end of the line. Thus, the reverse bearing of the line AB, Fig. 4, is BA, or the angle S′BA ; that is, South 35° East, or S. 35° E.

Sometimes the term **Course** signifies the **Bearing** and **Distance** of a line taken collectively.

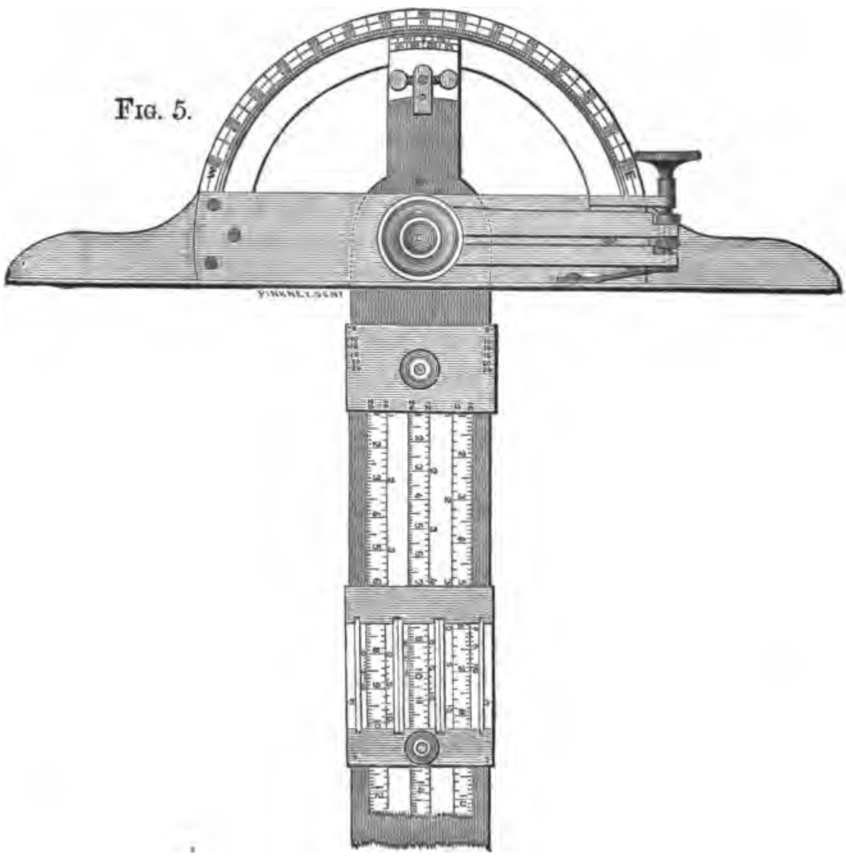
14. **Latitude** is the distance measured on a given meridian, between two parallels of latitude. It is called also **Northing** or **Southing**; or, **Difference of Latitude.** Thus AC, Fig. 4, is the *Northing* of the course AB ; and BC′ is the *Southing* of BA.

15. **Longitude** or **Meridian distance** is the distance measured on a parallel of latitude, of any point from a given meridian. Thus, in Fig. 4, the longitude of the course AB ; that is of the point B, is BC.

16. **Double Longitude** is the sum of two adjacent meridian distances ; being always either the base of a right angled triangle, or the sum of the parallel sides of a trapezoid. Thus, the double longitude of the right angled triangle AB*b*, Fig. 25, Art. 54, is B*b* ; and that of the trapezoid *b*BC*c*, is B*b*+C*c*.

THE TRIGONOMETER.

Fig. 5.



(2.) THE PARTS AND THEIR COMBINATIONS.

THE TRIGONOMETER combines in one machine the **Protractor, Draughting Rule,** and **Sliding Vernier Scale-Plate.**

(3.) **The Protractor** of the usual semicircular form, is made of German silver, firmly soldered to a frame of the same metal or of brass, whose front side formed into a double lip projecting upwards and downwards is called the *base* of the instrument. It is graduated to 70° on each side of its zero or meridian line; the divisions being made to half degrees. The reading of the divisions is *double;* the *inner* reading giving the *angle,* and the *outer* the *complement* of the same angle.

The *zero line* of the inner reading is marked N, signifying *meridian,* whether North or South. On the extreme right is the letter E, denoting *East;* and on the left W, denoting *West.* The divisions between N and E, indicate North or South so many degrees *East.* Those between N and W, indicate North or South so many degrees *West.*

(4.) **The Rule** consists of a steel plate 20 to 30 or more inches in length according to its specific use, two inches broad, and about 3-40ths of an inch in thickness.

The Rule is attached to the center of the protractor by means of a *pivot* and *socket,* which form their common axis.

(5.) To this pivot is attached a **Vernier** of German silver, adjustable (with reference to the Rule, to which it also is attached), by means of adjusting screws at its sides. The vernier corresponds with the divisions on the protractor, carrying its reading to *minutes* of a degree.

On the under side is a milled **Thumb-screw** *for clamping the vernier.*

(6.) In the *first class* instruments there is attached to the pivot a **Tangent Clamp.** This is tightened at pleasure by a screw and nut connected with it; but it needs never to be clamped only with sufficient tightness to secure motion to the vernier and Rule, by means of the milled thumb-screw and spring which act against it; very seldom requiring change. The screw and nut may be varied if necessary by pressing the

end of the screw-driver against the teeth of their milled edges. To produce the *delicate motion* necessary in setting the instrument at the exact angle to minutes of a degree with *expedition* and *nicety*, the tangent fixture is very convenient; the vernier having been previously brought very *near* the required angle by hand.

(7.) **The Scale-Plate** consists of a plate of German silver an inch and a half wide, about the 16th of an inch thick, and usually 15 to 20 inches in length. This plate has graduated upon it *six* decimal scales of different units*, according to one of the following systems, viz.: First, 10-8ths, 10-10ths, 10-12ths, 10-16ths, 10-20ths, and 10-24ths of an inch. This may be called the INCH SYSTEM, each of these several fractions constituting a *unit* of its *own* scale. Second, the MIXED SYSTEM; whose units are *inches* and *fourths* of an *inch; 10ths*, and *20ths of a foot; centimetres* and *double centimetres.*

(8.) There are connected with the scale-plate TWO GUIDES; one of them fastened to the plate, and the other movable, called ATTACHED and VERNIER GUIDES. The former has a knob connected with it, for convenience in sliding the plate on the rule. The latter guide has a knob for sliding it on the Scale-Plate. It consists of two side pieces of brass or German silver, connected together with rivets by two cross-pieces of the same metal, at right angles with the latter pieces. These side pieces are so milled as to form on each side of the guide a *lip* for guiding the Scale-Plate as it slides on the Rule. The cross-pieces are so milled as to receive the same number of *verniers* as there are scales marked on the Scale-Plate; each reading to the *10th part* of the subdivision, or hundredth part of the corresponding unit. By a little practice all the verniers except the $\frac{1}{4}$ inch unit, are easily read to the 1000th of the corresponding unit.

When the two guides are brought snugly together, their *common line of contact constitutes the unit line of all the scales.* And the distance from this line when the guides are in contact,

* For the use of some architects *duodecimal* scales are substituted for *decimal.* For the convenience of students, and for very small work, a smaller instrument is sometimes made with *four* scales instead of *six.*

or from a straight line joining the touching points of the vernier guide when apart, to the zero line of each vernier, is equal to either the corresponding *unit, half unit,* or *double unit* of the several scales.

Hence the *units* and *tenths* are read *on the Scale-Plate,* being indicated by the figures and subdivision lines which lie between the guides. The *hundredths* and *thousandths* are read directly *on the verniers.*

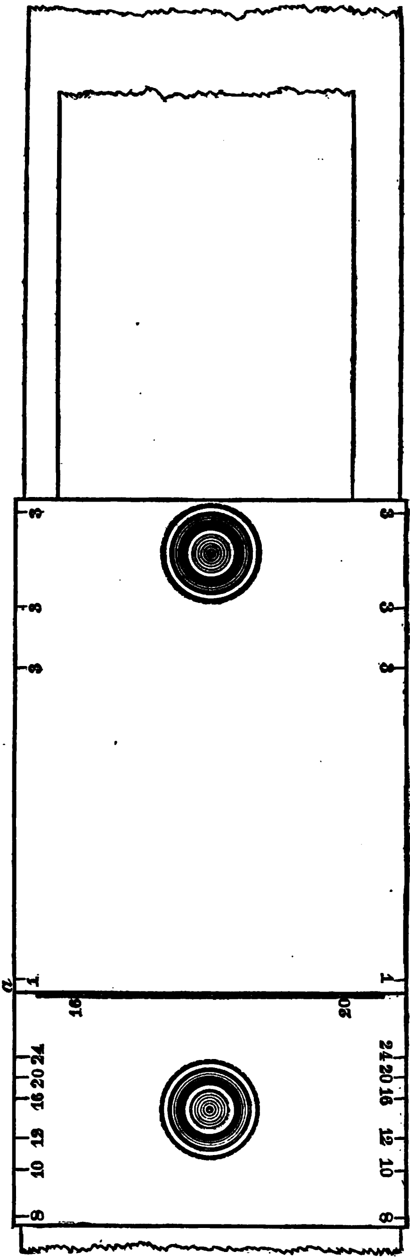
It will be seen, therefore, that there are *four points* on the Scale-Plate which may be termed *terminal* or *reading points.* When the guides are in contact, as in Fig. 6, these points are in the *unit line* of the Scale-Plate, and are reduced to *two,* viz.: *one on each border of the Rule.* But when the guides are slid apart, as they always are for measuring, or laying off distances, these points are *four,* viz.: *one on each border of the Rule,* and in the *terminal line* of the two guides.

(9.) On each side of the Vernier Guide there is a fine, knife edge GROOVE, close to the unit line; the two constituting a pair, or set, called *unit grooves.* On each side of the *Attached Guide* there are also *six other sets of grooves* like the former, called *zero grooves.* And when the guides are brought in contact, the first set, (that is, those next to the unit line, according to the "Inch System,") is 10-24ths of an inch from the unit grooves. The second set is 10-20ths ;* the third set, 10-16ths; the fourth set, 10-12ths; the fifth set, 10-10ths; and the sixth set, 10-8ths of an inch distant from the zero grooves. According to the "Mixed System," the first set of zero grooves is 10-40ths of an inch from the unit grooves; the second set one centimetre; the third set, one 20th of a foot; the fourth set, 2 centimetres; the fifth set, one inch; the sixth set, one 10th of a foot from the unit grooves.

Besides these, there are on the *Vernier Guide three other sets* of grooves, distant *two, four,* or *eight units* from the unit grooves. They are designated by the figure 3, to denote that they are severally *three units* of the coarser divisions distant from their *corresponding zero grooves.* The object of these

* Sometimes 10-40ths instead.

FIG. 6.



Outline of the Scale-Plate.

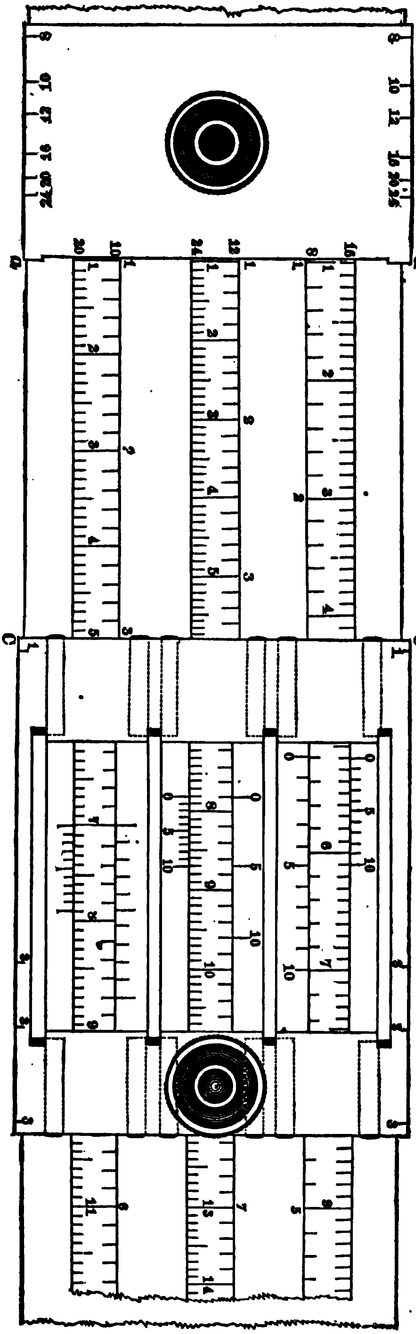
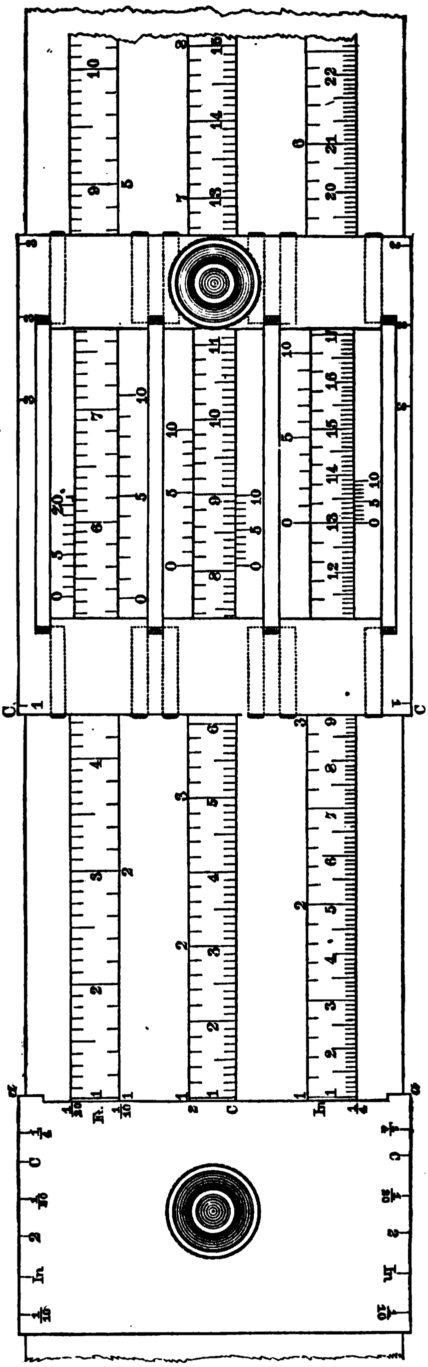


FIG. 7.

Scale-Plate with the "Inch System."

Fig. 8.



Scale-Plate with the "Mixed System."

last named grooves is to give greater available length to the Scale-Plate in extreme cases.

These explanations may be illustrated by the annexed engravings. Fig. 6, p. 20, shows the guides *in contact.* Fig. 7, p. 21, shows them *apart.* These figures illustrate the "*Inch System;*" Fig. 8, p. 22, the "*Mixed.*"

The *Grooves* cannot be represented in the engravings. Their *relative position* merely is indicated by the *short marks* shown on the extreme edge of the guides connected with the figures 8, 10, 12, &c. (Figs. 6 and 7), and $\frac{1}{10}$ In. 2, &c. (Fig. 8).

(**10.**) *a, a,* Fig. 6, and *a, a, c, c,* Figs. 7 and 8, are the *terminal* or *reading points;* being the *intersecting points* which the terminal lines of the guides make with the two borders of the Rule. The use of these points constitutes the First Method of Reading and Measuring. When this method is used for *laying down* distances, there is to be a *correction* of an error at the two terminal points, equal to the semidiameter of the prick used in marking distances; but in *measuring* distances no correction is necessary.

(**11.**) The Second Method, however, is altogether preferable. This consists *in using the grooves as the terminal points.* The numeral figures 24, 20, 16, 12, 10, and 8, belonging to the *Inch System,* designate *zero grooves;* which are respectively *one unit distant* from the *unit grooves.*

The following are the letters and figures belonging to the *Mixed System:* $\frac{1}{4}$ and 1 In.; C. and 2C.; $\frac{1}{20}$ and $\frac{1}{10}$ ft. In. denotes *Inch;* C., *Centimetre;* and ft., *foot.*

The several grooves are used in connection with the scales having a *corresponding signature.*

The figures on the several scales are adapted to the *second method* of reading and measuring. Whenever, therefore, the *first* method is used, a *correction* of *one unit* will be necessary.

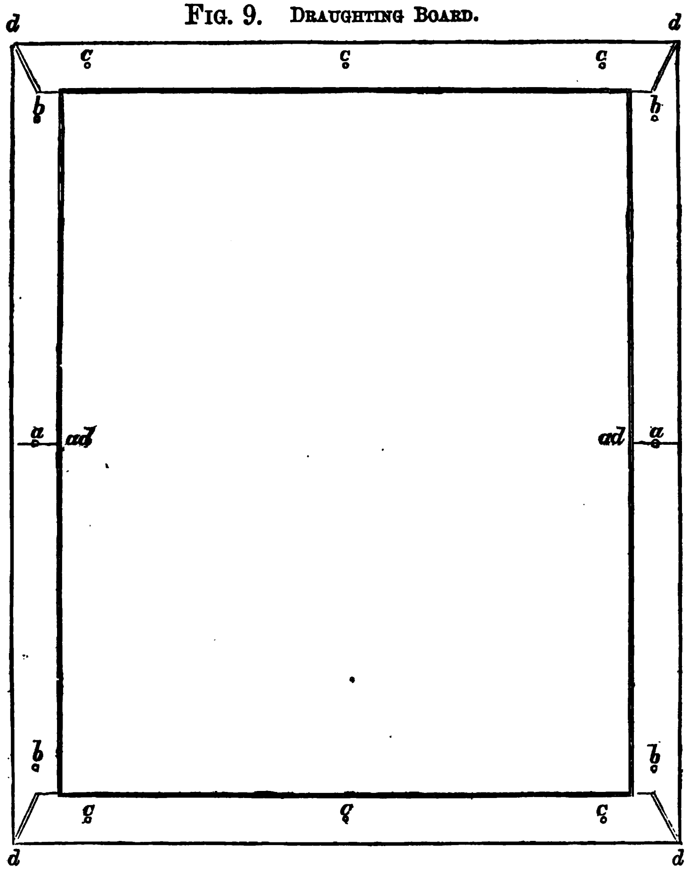
DRAUGHTING BOARD.

(**12.**) **A Draughting Board** or **Table** accompanies the Trigonometer, constructed as follows:

It consists of a *rectangular frame* usually 20 by 24 inches.

in width and length for the smaller size, and 25 by 30 inches for the larger size ; having a BOARD fitted to its inner border, removable at pleasure. Upon each of the four pieces forming the frame is fastened an *iron*, or *steel plate*, by means of screws attached to the plate which pass through the frame ; the plates

FIG. 9. DRAUGHTING BOARD.



being held firmly down by nuts underneath. A small wrench accompanies the table for turning the nuts. The edges of the plates project inward 1-8th inch or more beyond the inner edge of the frame ; thus forming a shoulder to which the board is fitted, and against which it is pressed by bars and springs underneath ; the upper surface of the board being in the same

plane with that of the metallic plates. An exact drawing of its upper surface is seen in the annexed figure, Fig. 9.

The Draughting Board may be used on a table three and a half or four feet high; or it may be used on a frame made for the purpose.

The little circles *a*, *a*, *b*, *b*, &c., Fig. 9., indicate the position of the *screws.* Those in the center of the *side-plates, a, a,* act as *pivots:* having no *play,* except what is necsssary to allow of a very slight rotary motion of the plates: while the screws *b*, *b*, *b*, *b*, pass through *enlarged* orifices in the frame, to admit of the same motion.

The orifices *c*, *c*, &c., in the *ends* of the table, are *all enlarged,* so as to admit (by simply *loosening* the nuts underneath), of a backward and forward motion in adjustment, and also to allow of expansion and contraction arising from change in the state of the atmosphere.

(**13.**) **Adjusting Lines.** At *exactly equal distances from the shoulders of the side-plates,* there are drawn across the plates, at *right angles,* and near their center, *fine, straight lines, ad, ad,* called ADJUSTING LINES. These are for *guiding the Rule in squaring the plates.*

DIRECTIONS FOR USING THE TRIGONOMETER.

(**14.**) **Adjustments.** Before attempting to use the Trigonometer, it is necessary that both the *instrument* and the *Draughting Board* should be brought into *exact adjustment.* In adjusting the former,

First, Bring the **0** of the Protractor Vernier to coincide exactly with its meridian, and tighten the clamp.

Secondly, Bring the base of the Trigonometer gently in contact with the Border of the Board, and slide it along *till the Rule comes in contact with the adjusting lines on the side-plates.*

Thirdly, Having observed how the Rule stands with reference to each of the adjusting lines, the instrument is to be inverted and observed again. If the Rule when brought into near coincidence with the lines occupies the *same relative position* with reference to them as before, the *Protractor Vernier*

is already adjusted. If otherwise, the adjusting screws must be *turned* one way or the other, till the relative position of the Rule and the adjusting lines is *the same,* with *either face* of the instrument *upward.* When this is accomplished, the little screws which fasten the vernier to the Rule (and which must be slightly loosened before the adjustment is made), are to be turned tight.

The adjustment of the vernier is made by means of a *bent screw-driver* that accompanies the instrument.

(15.) TO ADJUST THE DRAUGHTING BOARD.

Bring the Rule to coincide with the adjusting line on ONE *of the side plates,* and having loosened the nuts underneath, press outwards gently with the thumb or screw-driver by prying one end of the plate if necessary, at the points d, or d', till the Rule *coincides with* BOTH *the adjusting* lines, ad, ad; and clamp the nuts underneath. Do the same with the OTHER *side-plate.* Now, bring the *end-plates* (previously made loose), in *exact contact* with the *shoulders* of the *side-plates,* and clamp the nuts underneath. The adjustment is then finished.

(16.) TAKING ANGLES.

In all cases some one of the sides of the *Draughting Board* is assumed as the NORTH border. The opposite side then of course becomes SOUTH, the right hand EAST, and the left hand WEST.

If after the Trigonometer and Board are rightly adjusted, the former be set *at its Meridian,* and placed in contact with the *North* or *South* sides of the latter; all lines drawn by either edge of the Rule are MERIDIANS. And if the instrument be placed in contact with the *East* or *West* sides of the board, all lines drawn by either edge of the Rule are PARALLELS of LATITUDE.

In setting the instrument at any required angle, it is held by its graduated limb or frame, with *one hand*, and the Rule with the *other;* the graduated face being upward.

(17.) FOR ANGLES OF 45° OR LESS.

In obtaining angles from North to North 45° East or West, or their opposite angles, the INNER reading on the Protractor limb is used; and the instrument with the *graduated face* UPWARD, is applied either to the NORTH or SOUTH borders of the board.

It should however be particularly observed, that the *inner* reading *extends to* 60° *or more, on each side of the meridian;* also that the *outer* reading *begins at* 30° *or less on each side of the meridian.* Thus, *all angles between* 45° *and* 60°, *and their complements, on each side of the meridian,* may be obtained, or laid down, with EITHER FACE of the instrument UPWARD, as the case may require.

Cases will occasionally occur in which this facility of the instrument will be found a great convenience.

(18.) VERNIERS.

The Verniers both of the Protractor and Scale-Plate are *direct.* That is, they are read in the direction in which the figures of the graduated limb *increase.* If the figures increase *toward the right, the vernier is read toward the right.* If they increase *toward the left, the vernier is read toward the left.*

(19.) CASE 1. DEGREES, AND HALF DEGREES.

If the required angle consists of *degrees,* or *half degrees, without minutes,* the zero mark of the vernier is to be set at the required division mark on the protractor limb.

CASE 2. DEGREES WITH 15′ OR LESS.

If the required angle consists of *degrees with* 15′ *or less;* find the degrees as before. To obtain the *minutes,* carry the

zero of the vernier enough farther to make *that mark which corresponds with the given number of minutes, coincide exactly with some mark on the limb.*

CASE 3. DEGREES WITH 15′—30′.

If the required number of minutes is *between* 15′ *and* 30′; find the degrees as before; and commencing at the zero mark of the vernier, count the minutes on *that reading* of the vernier which is *adjacent* to the graduated limb, in the same direction in which the degrees have been read, whether to the right or to the left, till the 15′ mark is reached. Now turn the eye to the 15′ mark on the *other side* of the vernier and count, on the reading *farthest* from the limb, and in the same direction as before, till it reaches *that mark* on the vernier which *corresponds with the required number of minutes.* Then move the vernier till *the same mark* coincides with a mark on the limb.

CASE 4. DEGREES WITH 30′—60′,

To obtain any number of minutes between 30′ and 60′; carry the zero of the vernier to the required *half degree* as in *Case* 1. Then obtain the minutes precisely as in the *Second* and *Third Cases,* and add them to the 30′ before obtained.

(20.) ANGLES OF ELEVATION AND DEPRESSION.

When the Trigonometer is used for laying down or measuring *Angles of Elevation or Depression* the line representing the *horizon* becomes the *meridian.*

Thus, when the instrument is set at zero, and brought in contact with the *North* or *South* sides of the Board; all lines drawn by the Rule will be *horizontal lines.* And when the instrument is considered set at 90°, and brought in contact with the *East* or *West* sides of the Board; all lines drawn by the Rule will be *vertical lines.* Angles of Elevation or Depression are then read, *North* or *South* so many degrees *East* or *West,* as the case may be.

TAKING DISTANCES.

(21.) In Determining Distances first ascertain which of the six scales will give a sketch or plot of the desired dimensions; or, of dimensions corresponding in size with the sheet of drawing paper attached to the Draughting Board.

But if none of the scales will produce the right dimensions, then *multiply or divide* the several given distances by any convenient integral or fractional number, as 2, 10, 100; 2-10ths, 3-10ths, 4-10ths, &c. In practice, however, rarely will a case occur in which the numbers 2, 10, or 100, will not accomplish the purpose. For it will be readily seen that the use of these three numbers will add to those marked on the Scale-Plate, at least *thirty new scales.*

In laying down distances, the *units* may represent *feet, yards, rods, chains, or any other quantity.* But in laying down the distances of a *survey* it is generally much the most convenient method to consider *chains units.*

(22.) To Obtain a given Distance. Hold the Scale-Plate by the *Attached Guide* or its knob, with the left hand, and with the right, *slide the Vernier Guide* by its knob* to the required distance on the Scale-Plate; reading the *units* and *tenths* at the left edge of the latter guide, on such one of the scales as has been selected for the case on hand. The *hundredths* and *thousandths* are read on the corresponding vernier. The hundredths are directly indicated by the figures on the vernier. The thousandths are determined by *judging how great a portion of a hundredth* should lie between the nearest less hundredth mark, and the corresponding mark on the scale, in order to *equal the given number of thousandths.* A little practice will render this easy and certain.

(23.) LAYING DOWN ANGLES AND DISTANCES.

The Protractor Vernier being set and clamped at the required

* It is sometimes found convenient, in setting the Vernier Guide, to hold the plate in both hands with the thumb nails resting against the two ends of the Guide. In this way a little practice will enable the operator to set it with *expedition* and *great accuracy.*

angle, according to the directions already given, and slid on the board till the Rule is brought close to the starting point, or dot on the paper, the Scale-Plate with its Vernier Guide set at the required distance, is then placed on the Rule and slid till the *Terminal point* of either the Attached or Vernier Guide (as the case may require), coincides with the same dot: when a second dot is made with a needle pointed prick, in the corresponding terminal point of the *other Guide*, at the other end of the given distance. The Scale-Plate is then carefully lifted from the Rule, and the two dots are connected when necessary by fine lines drawn with a draughting pen, or a hard pencil sharpened with a flat point.*

The Protractor Vernier is now set at the next required angle, the instrument brought to its proper position, close to the second dot; the vernier guide of the Scale-Plate set at the next required distance, and the same operation repeated as before, and so on, till the outline of the sketch or survey is finished.

MEASURING ANGLES AND DISTANCES.

(24.) For MEASURING *angles and distances already laid down,* the operation is REVERSED in the following manner, viz.: With the base of the instrument in contact with the border of the table, the *Rule* is brought to *coincide* with the *given points* on the paper; and the Protractor Vernier having been clamped, the Rule is slid back a little, and the *grooves of the guides* brought to coincide with the *same points.* The angle is then read off on the Protractor, and the distance on the Scale-Plate.

Whenever it is required to measure an angle or a distance *separately,* the Protractor with the Rule for the former, or the Scale-Plate for the latter, as suits the case, is to be used *separately.*

* Faber's drawing pencils, No. 4, answer this purpose very well. They may be nearly sharpened with a knife and fine sand paper, but should be finished on *fine emery paper;* the pencil being held in the hand, and the paper lying flat on a table.

USE OF DOTS.

(**25.**) It is often a saving of time, preserves the paper from being blemished by pencil marks, and ensures a more reliable result, to indicate in a preliminary operation the angular points *by mere dots with the prick*, surrounded by temporary little circles made lightly with a common soft pencil.

By this method, too, as soon as the outline is finished, and the starting point reached, *any error* existing in the data is instantly detected, and in most cases readily located and corrected without drawing a single line.

The operation may then be repeated; the outline being at the same time permanently drawn, if necessary, with a draughting pen and India ink. As soon as the second operation is finished, the soft pencil marks may be erased with India rubber.

(**26.**) SPECIAL DIRECTIONS.

1. In all cases where great exactness is required, a *Microscope* of three or four inches focal length should be used for laying down and measuring angles and distances; and it should be so constructed that it can be used without the aid of the hand. To meet this want a Microscope accompanies every Trigonometer, having a flat, brass ring attached to its case. From this ring project *three lips*, the two upper ones forming *spring hooks*, and the lower one a *spring hold* for attaching it to the *eye of a spectacle frame*, either with or without glasses as may be desired. The lips may be tightened or loosened at pleasure by bending with the thumb to fit a light or heavy frame. The microscope is put into its place by first slipping the *lower side* of the spectacle eye under the lower broad lip; next sliding it down far enough to allow the hooks to slip over its top; and then sliding it upwards, till the center of its eye coincides with the center of the Microscope.

2. An angle may be *tested* in the following way. First, lay it down with the Trigonometer in its usual or *normal position*.

Then calling the letter *E*, on the Protractor limb, *West*, the

letter *W*, *East*, and setting the vernier at the corresponding angle, *invert the instrument*, lay the angle down as before, or *read* it, as the case may need. If there should be found a variation from the angle as before read, or laid down, take the mean of the two as the *true angle* or *reading*.

3. In using the Trigonometer, great care is always necessary to see, at the moment an angle is laid down or measured, that its *lip* or *base is in exact and close contact with the border of the Board.*

4. When the distances to be laid down or measured are *less than a unit*, the *first method* of reading and measuring, (Art. 10.,) may be employed. But for *laying down* this class of distances the following is generally a more convenient method, viz.:

With the *reverse bearing*, the two guides being in contact, make a dot at the distance of ANY UNIT from the dot last made; then from this point, lay off the *given distance plus* THE SAME UNIT, in the line of the *direct bearing*. This gives the required point.

5. It is essential to accuracy in using the Trigonometer that the *prick* used for making dots should have a strictly *fine, needle point*, that the *dots* made should be only *large enough to be seen*, and that the *pencil* used should be *hard*, and brought to an *exact knife-edge*.

(27.) PREPARING THE PAPER, ETC.

In order to have the paper lie smoothly upon the board, it must be *moistened*. This is done by first laying it flat upon the board, and applying to it a large, soft sponge, wet with clean water, partly wrung out. The paper should be held firmly to the board with one hand, and with the other the sponge should be drawn from the center outwards till it is thoroughly moistened.

(28.) FIRST METHOD OF ATTACHMENT.

As soon as the paper is moistened, it is spread smoothly upon the board with the wet side down. The Trigonometer

without the Scale-Plate is then laid upon the paper, and slid so near one of its edges as to leave a border of half an inch or more to be turned up against the side of the Rule, to receive a coating of gum arabic, paste, or glue. The border is then pressed down with the Rule, or fingers, or a clean cloth; and if the paper has been properly moistened, it will readily adhere to the board. As soon as the first side is smoothly attached, the Rule is to be slid with care to the opposite side of the paper; and after that side is disposed of like the first, a third and fourth is successively taken.

The operation of sticking down should be performed expeditiously, so as to prevent the paper becoming too dry to adhere well.

(29.) SECOND METHOD.

There is another method of fastening the paper, which in most cases is altogether preferable to the one just described.

The board is first taken out of the frame, and laid in a convenient place for receiving the paper; which, cut of the same size as the frame, is then moistened and placed upon the board in the manner described (Art. 28). The wrinkles may be removed by raising up first one side, then another, and pressing it down from the center outwards by passing the hand carefully over it, or a clean, soft piece of cotton cloth. After cutting off a piece at each corner, so that at those points the paper may be turned smoothly over, the frame is laid upon it, and fastened in its place by the bars underneath.

(30.) THIRD METHOD.

Sometimes when one is in haste, it is sufficient merely to apply glue at the corners of the paper, and at several points on each side, and stick it down at once without moistening.

This will do very well, provided, before using, the board has been lying in a cold room—to be used in a warm one. In this case however, it will be necessary to apply weights to the

several points of attachment for a few minutes, till the glue is dry.

This method is also convenient when it is required to draw so large a plot that but three sides only of the board can be used at once. For, by moistening the places of attachment after one end of the plot is finished, the paper may be *detached, shifted* across the board and *re-attached* for finishing the other end. But before shifting, a meridian or parallel of latitude, or a line of some other definite course should be drawn with the hard pencil near the unfinished side of the board, so that in the two positions the courses may be made to correspond with each other.

It is often desirable in plotting surveys to attach the paper to the board in such manner as to *make the paper coincide with some one of the sides of the plot* to be drawn. In this case the *second method* of attachment can not be used, as the paper must lie on the board to some extent *diagonally*.

(31.) On the other hand, the necessity of attaching the paper to the board *obliquely* is avoided by CHANGING THE BEARINGS *of the sides* of the field to be plotted, in such manner as to make *that side* which we wish to coincide with one of the edges of the paper, either a MERIDIAN or a PARALLEL OF LATITUDE, as may be the more convenient. This may be done by the following RULE, which should be made *perfectly familiar* to every one who uses the Trigonometer.

RULE.

To Change the Bearings. FIRST.—Suppose the field surveyed, or a map of it *to make a part of a revolution on its axis,* and call the desired amount of angular motion the **Changing Angle.**

SECONDLY.—If the *given angle* is in the *same* or the *opposite quadrant* with the *changing angle, take their* SUM ; but if in *adjacent* quadrants, *take their* DIFFERENCE for the CHANGED BEARING.

THIRDLY.—If the *sum exceeds* 90°, *subtract it from* 180°;

and both in this case, and whenever in taking the *difference,* THE CHANGING ANGLE IS GREATER THAN THE GIVEN ANGLE, the changed bearing must be placed in *that adjacent quadrant toward which the field is supposed to revolve.*

The rule may be illustrated by the following examples :

EXAMPLE 1.—Let the changing angle be N. 20° W.

1st Course,	N. 18° W.	∴ By case 2,	18° + 20°	=N. 38° W.	=Ch. Bg.
2d, "	N. 75° W.	∴ " " 3,	180° −(75°+20°)	=S. 85° W.	= "
3d, "	S. 5° 45′ E.	∴ " " 2,	5° 45′+ 20°	=S. 25° 45′ E.	= "
4th, "	S. 87° E.	∴ " " 3,	180° −(87°+20°)	=N. 73° E.	= "
5th, "	N. 4° 10′ E.	∴ " " 3,	20° − 4° 10′	=N. 15° 50′ W.	= "
6th, "	N. 56° 3′ E.	∴ " " 2,	56° 3′− 20°	=N. 36° 3′ E.	= "
7th, "	S. 2° 48′ W.	∴ " " 3,	20° − 2° 48′	=S. 17° 12′ E.	= "
8th, "	S. 78° W.	∴ " " 2,	78° − 20°	=S. 58° W.	= "

EXAMPLE 2.—Let the changing angle be N. 44° 50′ E.

1st C'se,	N. 39° 15′ E.	∴ By case 2,	39° 15′+ 44° 50′	=N. 84° 5′ E.	=Ch. Bg.
2d, "	N. 48° E.	∴ " " 3,	180° −(48°+ 44° 50′)	=S. 87° 10′ E.	= "
3d, "	S. 25° 16′ W.	∴ " " 2,	25° 16′+ 44° 50′	=S. 70° 6′ W.	= "
4th, "	S. 62° W.	∴ " " 3,	180° −(62°+44° 50′)	=N. 73° 10′ W.	= "
5th, "	N. 43° W.	∴ " " 3,	44° 50′− 43°	=N. 1° 50′ E.	= "
6th, "	N. 74° 14′ W.	∴ " " 2,	74° 14′− 44° 50′	=N. 29° 24′ W.	= "
7th, "	S. 16° E.	∴ " " 3,	44° 50′− 16°	=S. 28° 50′ W.	= "
8th, "	S. 60° 30′ E.	∴ " " 2,	60° 30′− 44° 50′	=S. 15° 40′ E.	= "

In the *first* example, to a person standing at its center the field is supposed to revolve from *right to left*. In the *second* example the change is from *left to right*.

(32.) PREPARING INK.

In *preparing India Ink for drawing lines with the draughting pen*, the ink should be rubbed, with a little soft water, upon a piece of smooth marble, ground plate glass, or other hard, silicious, or earthern substance till it is sufficiently thickened to produce a jet black mark by twice or thrice drawing it. If after standing for a time it becomes too thick to flow freely, it should be thinned by the addition of a little water.

In order to insure a free, uniform deposit of the ink, and leave a smooth mark, it is necessary frequently to remove the ink from the pen, clean it with a piece of moistened, soft cotton cloth, or sponge, and replenish it with fresh ink. For the same reason, before drawing any *fine* line, the point of the pen should be touched to the moistened cloth or sponge.

Special care with the pen and ink is needful in drawing the *hair lines* so common in using the Trigonometer.

It should never be forgotten, however, that *no draughting pen* can be made to work *with the accuracy* or to *take the place* of the HARD, KNIFE-EDGE PENCIL. Hence it is *indispensable* that the latter should be *always kept with a smooth, sharp edge.*

SECTION II.

RULES FOR SOLVING WITH THE TRIGONOMETER, THE PROBLEMS OF PLANE TRIGONOMETRY.

RIGHT ANGLED TRIANGLES.

CASE I.

(**33.**) *The hypothenuse and an angle being given, to find the* BASE *and* PERPENDICULAR.

RULE.

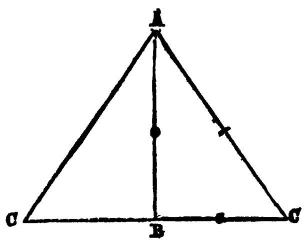
1. From an assumed point, with the Trigonometer draw a *meridian* (Art. 16), of indefinite length *for the perpendicular.*

2. With the Vernier Guide set at the given distance or length of the hypothenuse, lay off from the assumed point, on either side of the meridian, the *given angle* and *distance.*

3. From the other end of the hypothenuse draw a *parallel of latitude* meeting the meridian already drawn. The length of the base and perpendicular are then obtained by *direct measure* with the Scale-Plate.

Example 1. If the hypothenuse AC, Fig. 9, be 95 chains, and the angle at A, 35° 10′; what is the length of the sides AB, and BC?

FIG 9.



First, from the point A, draw the meridian AB, of indefinite length.

Next, with the bearing South 35° 10′ East (or West), and the distance 95, draw AC.

Lastly, with the instrument set at 90°, that is, *at right angles,* draw as a parallel of latitude from the point C, the line BC.

By applying the Scale-Plate to the required sides, the base

BC is found to be 54.72 chains, and the perpendicular AB 77.66 chains.

Ex. 2. The distance from the eye of an observer to the top of a church spire is 125 feet; the angle of elevation to the same point is 27° 15′. What is the horizontal distance to the center of the spire on a level with the observer's eye, and what is the height of the spire above the same point?

According to the directions given (Art. 20), read the given angle of elevation North, *or South*, 27° 15′ East, *or West*, as is the more convenient.

Ans. { Horizontal distance, 111.1 feet. Height, . . 57.23 "

CASE II.

(34.) The *hypothenuse* and *one leg* being given, to find the *angles* and the *other leg*.

RULE.

1. From an assumed point draw a *meridian* corresponding in length with the *given leg*.
2. From *one end* of this leg and at *right angles* with it, draw the *other leg of indefinite length*.
3. Set the Vernier Guide at the given hypothenuse distance; at the point where this distance measured from the assumed point meets the line of indefinite length, make a dot, and connect it by a line with the assumed point; then measure the angles and required leg according to (Art. 24).

Ex. 1. The hypothenuse being $11\frac{1}{2}$ miles, and the base 7 miles; required the perpendicular, and each of the angles.

FIG. 10.

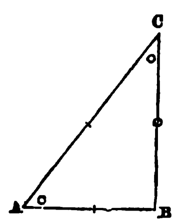
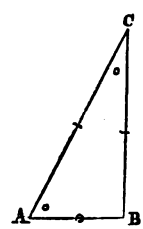


FIG. 11.



First, draw the base AB, Fig. 10, 7 units long (Art. 7), as a meridian.

Next, at right angles to AB draw BC, of indefinite length.

Then from the point A, measure off $11\frac{1}{2}$ units; and at C, where the terminal point of the Scale-Plate meets the indefinite line BC, make a dot. Having joined A and C, ascertain the bearing of the line, which is the angle A. This subtracted from 90° (Art. 1, Def. 11), gives the angle C. But as it is the *complement* of the angle A, it is *read off* on the *complement reading* of the protractor limb instantly, *without subtracting:* and the leg BC, is measured with the Scale-Plate.

Ans. { Angle A = 52° 30′
" C = 37° 30′
Perpendicular = $9\frac{1}{8}$ miles, nearly.

Ex. 2. Given the hypothenuse Fig. 11, 54 leagues, and the perpendicular 48 leagues; what are the angles and base?

Ans. { Angle A = 62° 44′
Base AB = 24.74 leagues.

CASE III.

(**35.**) The *angles* and *one leg* being given, to find the *hypothenuse* and the *other leg.*

RULE.

1. As in the previous Case draw the *given leg as a meridian.*
2. From *one end* of this line and at *right angles* draw the *other leg of indefinite length.*
3. From the *other end* lay off the *adjacent given angle.* Then measure the required sides with the Scale-Plate.

This, and the following Case are so plain that they need no illustration.

Ex. 1. Given the base 60, and the adjacent angle 47° 12′; what is the length of the perpendicular, and the hypothenuse?

Ans. { Hypothenuse = 88.3.
Perpendicular = 64.8.

Ex. 2. Given the perpendicular, 193.6, and the angle opposite the base 47° 51′; required the hypothenuse and the base.

Ans. { Hypothenuse = 288.5.
Base = 213.9.

CASE IV.

(**36.**) The *base* and *perpendicular* being given, to find the *hypothenuse* and the *angles*.

RULE.

Draw *one of the legs as a meridian;* then from *one end* of the line and at *right angles* to it draw the *other leg.* Having joined the *unconnected ends* of the two lines, the angles and hypothenuse may be measured as in the previous Cases.

Ex. 1. If the base be 640, and the perpendicular 480, what are the angles and hypothenuse?

Ans. Hypothenuse = 800. Angle at base = 36° 52′.

Ex. 2. Given the base 32, and the perpendicular 26, to find the hypothenuse and the angles.

Ans. Hypothenuse = 41.23. Angle at base = 39° 6′.

OBLIQUE ANGLED TRIANGLES.

CASE I.

(**37.**) *Two angles* and a *side* being given, to find the *remaining angle,* and the other *two sides.*

The *third angle* is found by merely subtracting the sum of the two angles from 180°. (Def. 11, Art. 1.)

RULE.

Draw the *given side* as a *meridian.* Then at *one end* of the line lay off *one of the angles,* and at the *other end,* on the same side of the line, lay off the *other angle.* The point of intersection of the two latter lines will determine the length

of the required sides. These may be measured as in the former Cases.

Ex. 1. In the triangle ABC, Fig. 12, the side AC is given 32 chains, the angle A, 56° 20′, and the angle C, 49° 10′, to find the angle B, and the sides AB, and BC.

Fig. 12.

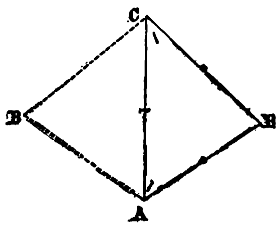
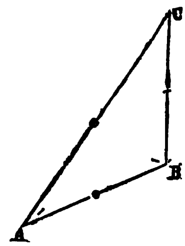


Fig. 13.



Draw AC, as a meridian. Then with the Trigonometer set at N. 56° 20′ E. (or W.), draw AB. Then with it set at S. 49° 10′ E. (or W.), draw CB. Now with the Scale-Plate measure the lines AB and BC, and the work is done.

Ans. AB = 25.13 chains, and BC = 27.64 chains.

Ex. 2. Given BC, Fig. 13, = 98, the angle A = 31° 21′, and the angle B. = 114° 24′; required the sides AB and AC.

Ans. AB = 95.12, and AC = 162.3.

CASE II.

(**38.**) *Two sides* and an *opposite angle* being given, to find the *remaining side*, and the other *two angles*.

RULE.

1. Draw as a meridian *that given side which is adjacent to the given angle.*

2. At *one end* of this line, lay off the given angle by a *line of indefinite length.*

3. Measure off with the Scale-Plate from the *other end* of the *former to the latter* the remaining given distance; at the point of intersection *make a dot,* and connect by a line the two points. Applying the Trigonometer, the required angles and distances are read off as in the previous Cases.

Ex. 1. Given the angle A, Fig 14, = 35° 20′, the opposite side BC = 50, and the side AC = 70: to find the remaining side and the other two angles.

In examples under this rule like this and the following, in which *the side opposite the given angle is smaller than the other given side, the length of the required side is ambiguous.* For, from the angle opposite to the required side there may be drawn to that side produced, *two straight lines equal to one another,* viz.: one on each side of the perpendicular.

Thus the sides CB and CB′, Fig. 14, are equal. Hence the required side is either AB, or AB′; each of which will answer the conditions of the question equally well.

It should however, be remarked, that very rarely will a case occur, in practice, in which there will not be some circumstance that will determine which of the two values of the required side is the true.

In Ex. 1. First, draw AC as a meridian. Then with the bearing of N. 35° 20′ E. (or W.), draw AB of indefinite length.

Next, measure off from the point C, 50; that is, the side CB, or CB′; making a dot at B, or B′.

Lastly, measure the side AB, or AB′, and the angle C, as in the previous Cases.

Ans.

The	angle	B	=	125° 56′.
"	"	B′	=	54° 4′.
"	"	C	=	18° 44′, or 90° 36′.
"	Side	AB	=	27.76.
"	"	AB′	=	86.45.

FIG. 14.

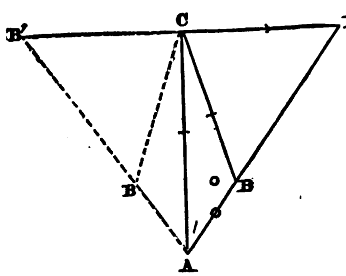
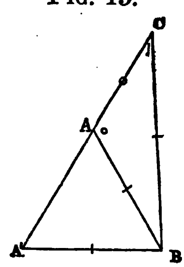


FIG. 15.



Ex. 2. Given the angle C, Fig. 15, $= 33° 21'$, the side BC, $= 95.12$, and the side AB $= 60$; to find the angles A and B, and the side AC.

Ans.

The angle A, *i. e.*	BAC,		$= 119° 22'$.
"	"	ABC,	$= 27° 17'$.
"	"	A′	$= 60° 38'$.
"	"	A′BC,	$= 86° 1'$.
"	Side	AC,	$= 50.03$.
"	"	A′C	$= 108.9$.

CASE III.

(**39.**) *Two sides* and the *included angle* being given, to find the *remaining side*, and the *other two angles.*

RULE.

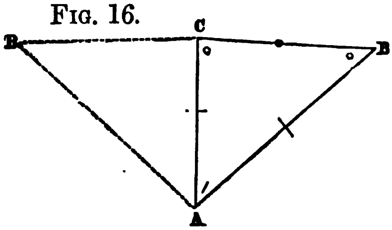
Draw one of the given sides as a *meridian.* Next, from *one end* of this line, with the *distance* of the *other given side,* lay off the *given angle.* Then having joined the unconnected ends of the two lines, measure as in the former Cases, the required side and angles.

Ex. 1. In the triangle ABC, Fig. 16, there are given AB $= 128$, AC $= 90$, and the angle A $= 48° 12'$; to find the angles B and C, and the side BC.

Draw the side AC (or AB), as a meridian. Next, with the distance AB (or AC), from the point A, lay off the given angle N. 48° 12′ E. (or W.).

Now, holding the Trigonometer base firmly in contact with the table border, bring the edge of the Rule to coincide exactly with the points B and C; and having clamped the instrument, read off the angle S. 87° 11′ E., (or W.;) which is the angle C.

FIG. 16.



Lastly, laying the Scale-Plate upon the Rule, measure the distance BC, $= 95.54$. The points B and C, may be joined if necessary. The angle B $= 180° - (A + C) = 44° 37'$.

REMARK.—The extreme ease, as well as accuracy and quickness, with which this operation is performed, is a fair illustration of the great advantage of the use of the *Protracting Trigonometer*, over any method hitherto employed in the solution of trigonometrical problems.

Ex. 2. Given, the side *b*, 58, the side *c*, 67, and the included angle A = 36°; to find the angles B and C, and the side *a*.

Ans. Side $a = 39.57$. Angle C = 84° 30′. Angle B = 59° 30′.

Ex. 3. Given the side *a*, 109, the side *b*, 76, and the included angle B, 101° 30′; to find the angles A and C, and the side *c*. *Ans.* Angle A = 30° 57′. Angle C = 47° 33′.
Side AC = 144.8.

CASE IV.

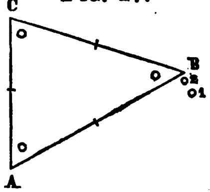
(40.) The *three sides* being given to find *the angles*.

RULE.

Draw *one* of the three sides *as a meridian*. Then from *one end* of this line measure off, with the Scale-Plate, *one* of the *remaining sides*, and from the *other end* measure off the *other side*. The point where these last two lines meet, is the third angular point. The angles can then be measured with the Trigonometer as in the former Cases.

The third angular point is generally determined by approximation, in the manner shown in the following example.

FIG. 17.



Ex. 1. If the sides of the triangle ABC, Fig. 17, are AB = 104, BC = 96, and CA = 78; what are the angles?

First draw as a meridian some one of the sides, say CA = 78 units; feet, chains, or any quantity desired. Then from the point A, with the Scale-Plate, lay off the distance AB = 104 units, as near 96 units from C, as possible; say at dot 1. Now measure from the point C toward 1, 96 units. It does not reach it, but falls at dot 2. Now re-measure AB, and dot 2 is found to be too near A. Make then a new dot just as far from C, if possible, as dot 2 is; say at the point B,

and re-measure CB. The point B is found to be nearly correct, within say the semi-diameter of the dot. Correct it slightly, draw AB and BC; then measure the angles A, B, and C.

Ans. Angle A $= 61° 43'+$, B $= 45° 42'$, C $= 72° 34'+$.

REMARK.—After a little practice a person of ordinary skill will obtain the precise angular point by making *one* or *two* proximate dots. Practically this fourth Case will be found as simple and accurate as the others.

Ex. 2. Given the three sides 634, 268, and 542; to find the angles. *Ans.* 58°, 97° 12′ and 24° 48′.

(**41.**) Although for the sake of perspicuity, illustration and system, *eight specific rules* have been given in this section, yet after the pupil has once *made himself familiar* with the use of the Trigonometer, nearly all with which he needs to burden his memory, is *first*, the general principle that, *in all examples connected with* ISOLATED TRIANGLES, ONE *of the* SIDES *must be made a meridian.* And *secondly*, that *of the six quantities connected with every triangle, viz.: the angles and sides*, at least THREE MUST BE GIVEN *in order to determine the* OTHER THREE; *of which* ONE *must be a* SIDE.

Bearing in mind these simple rules, any person of ordinary ingenuity, will readily devise a way to obtain the desired result.

It should also be recollected that in order to *insure accuracy* in the results the sketch or plot should be drawn on as *large a scale* as the board will conveniently allow.

(**42.**) In the foregoing rules, directions are given for *drawing lines* in solving the examples. But in practice the operations will be much more expeditious and satisfactory, and the results more accurate, by simply placing at the *angular points* very small *dots, without connecting them by lines*, (Art. 25.) Short ones, drawn with the knife-edge pencil, for designating the places of their intersection, are all that are necessary.

REMARK.—For further examples illustrating the rules, and for practical questions in the Mensuration of Heights and Distances, involving the use of the Trigonometer, the student is referred to *Stoddard & Henkle's* complete treatise on *Trigonometry* and *Land Surveying*, alluded to in the preface.

SECTION III.

METHODS OF DETERMINING AREAS.

(43.) PLOTTING.

THE first step in determining the area of a field by this system, after obtaining the data with chain and compass is, carefully to *plot its outline*, according to directions given (Art. 23).

It will facilitate the operation to write down the *angles* and *distances* of the several sides in *distinct columns*, placing before each its number, as in the following examples.

Ex. 1.	1.	N. 10° 15′ W.	5.65 chains.
	2.	N. 47° 10′ E.	7.83 "
	3.	S. 12° 52′ E.	8.20 "
	4.	S. 25° 0′ W.	3.98 "
	5.	N. 81° 38′ W.	4.93 "

In this and the following examples it is required to make the closing dot accurately coincide with the starting point. Whatever variation may appear will show the precise quantity and direction of any error made in plotting.

Ex. 2.	1.	S. 88° 15′ W.	17.75 chains.
	2.	N. 3° 5′ W.	9.46 "
	3.	N. 79° 20′ W.	8.11 "
	4.	N. 24° 18′ E.	15.22 "
	5.	East,	7.01 "
	6.	S. 51° 20′ E.	10.20 "
	7.	S. 15° 15′ W.	6.25 "
	8.	S. 29° 10′ E.	13.59 "

Ex. 3.	1.	S. 30° 0′ E.	27.02 chains.
	2.	S. 60° 3′ E.	28.00 "
	3.	N. 0° 10′ E.	20.01 "
	4.	N. 89° 55′ W.	7.54 "
	5.	N. 1° 2′ W.	14.16 "
	6.	————	——

Required the bearing and distance of the 6th side.

Ans. N. 83° 55′ W. 30.2 chains.

(44.) OMISSIONS IN THE FIELD DATA.

Ex. 4.	1.	S. 15° 47′ W.	——
	2.	N. 43° 5′ W.	4.68 chains.
	3.	N. 33° 0′ W.	8.66 "
	4.	N. 35° 20′ W.	6.48 "
	5.	N. 42° 42′ W.	5.41 "
	6.	N. 8° 47′ E.	7.95 "
	7.	S. 79° 0′ E.	3.20 "
	8.	S. 10° 38′ W.	1.96 "
	9.	S. 81° 4′ E.	8.05 "
	10.	N. 80° 46′ E.	——

Required the distances of the 1st and 10th sides.

Ans. 1st side, 26.75 chains; 10th side, 10.81 chains.

In this fourth example, it will be well to make the first end of the 2nd course the *place of beginning*. After all the sides except the *first* and the *last* are plotted by dots; with the reverse bearing of the former, and coinciding with the place of beginning, draw with the knife-edge pencil, a short hair line where it is presumed the remaining angular point will fall. With the bearing of the 10th side, and coinciding with the latter terminus of the 9th, draw another short line intersecting the former. The point of intersection will be the required angular point. The length of the required sides may then readily be ascertained with the Scale-Plate.

From these illustrations we derive the following rule for *supplying omissions in any two of the sides* of a field.

First plot the sides *whose data are complete*.

Next, from the *unconnected ends* of the terminal sides, lay off the given angles or distances of the *remaining two sides*. The *intersection* of the *two lines* will be the *required angular point*. The bearings or distances, or one of each, as the case may be, can then be measured with the Trigonometer; and lastly the survey may be *plotted anew* with the *complete* data.

Ex. 5.	1.	————	15.16 chains.
	2.	N. 50° 0′ W.	22.10 "
	3.	North,	18.83 "
	4.	N. 85° 0′ E.	35.65 "
	5.	S. 47° 0′ E.	29.02 "
	6.	S. 20° 30′ W.	—— "
	7.	N. 51° 15′ W.	26.47 "

Required the bearing of the 1st side, and the distance of the 6th.

Ans. 1st side, S. 45° 33′ W. 6th side, 23.81 chains.

In example 5th, First, plot the sides 2, 3, 4, 5, and 7; then from the unconnected end of No. 7, lay off the bearing of No. 6. Next, from the unconnected end of No. 2, and meeting this line, measure off the given distance of No. 1. Lastly, after ascertaining with the instrument the bearing of No. 1, and the distance of No. 6, plot anew the survey so far as may be necessary.

It is evident from this example that where two sides have omissions, if of the two given quantities, one be a *bearing* and the other a *distance*, the *former* should always be laid down *before the latter*.

Let it be particularly observed that it is unessential at *what angular point* in the outline the plotting is *commenced;* or whether the given courses are taken in *direct* or *reversed* order. The general rule is, *follow the order which is the most convenient.*

Ex. 6.	1.	S. 40° 30′ E.	31.80 chains.
	2.	N. 54° 0′ E.	—— "
	3.	N. 29° 15′ E.	2.21 "
	4.	————	35.35 "
	5.	N. 57° 0′ W.	20.90 "
	6.	S. 47° 0′ W.	31.30 "

Required the *distance* of the 2d *side*, and the *bearing* of the *4th*.

Ans. 2d side, 2.09 chains; 4th side, N. 28° 45′ E.

Ex. 7.	1.	S. 46° 10′ W.	10.51 chains.
	2.	S. 38° 24′ W.	8.21 "
	3.	N. 87° 40′ E.	7.92 "
	4.	S. 5° 45′ E.	18.10 "
	5.	———	15.12 "
	6.	N. 1° 16′ E.	40.08 "
	7.	West,	13.15 "
	8.	———	12.19 "

Required the bearings of the 5th and 8th sides.

Ans. 5th side, S. 76° 35′ E.; 8th side, S. 2° 25′ W.

Ex. 8.	1.	N. 7 25′ E.	7.00 chains.
	2.	N. 88° 52′ E.	16.50 "
	3.	N. 3° 10′ W.	11.82 "
	4.	S. 85° 40′ W.	23.07 "
	5.	N. 7° 15′ E.	9.18 "
	6.	N. 4° 56′ E.	12.02 "
	7.	———	8.41 "
	8.	S. 81° 30′ E.	30.48 "
	9.	S. 3° 55′ E.	14.00 "
	10.	———	32.03 "
	11.	N. 75° 4′ W.	30.10 "

Required the bearings of the 7th and 10th sides.

Ans. 7th side, N. 71° 22′ W.; 10th side, S. 18° 13′ E.

It will be desirable for every one learning the use of the Trigonometer, in part for the purpose of proof, and partly to perfect himself in plotting, to *re-plot* the foregoing eight examples, using the *completed* data.

(45.) TEST OF SECOND DOT.

When great nicety is required, it will be well after making the dot at the end of a distance, and removing the Scale-Plate

from the Rule, to slide the instrument back a little from the dots, and bring it gently in range with them the second time, before drawing the line, in order to test the correctness of the *dot last made.* If the two do not exactly coincide with the edge of the rule, the dot in question should be corrected with the prick point; special care being taken in drawing the line, that the pencil edge *accurately bisects* the two dots.

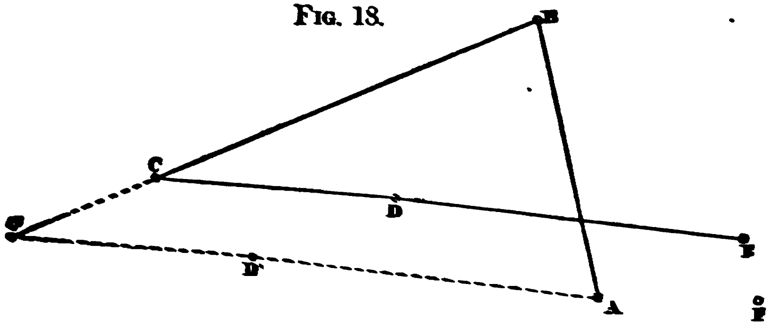
(16.) DETECTION AND CORRECTION OF ERRORS.

No method hitherto known can afford such simplicity, facility and certainty for detecting and correcting errors as are furnished by the use of the *Trigonometer.*

The following example will illustrate the process. Fig. 18.

Ex.	1.	N. 10° 0′ W.	18.00	chains.
	2.	S. 69° 50′ W.	26.20	"
	3.	S. 83° 10′ E.	15.25	"
	4.	S. 81° 6′ E.	22.24	"

FIG. 18.



In plotting the survey in this case, the point E does not fall on A, as it should. But by applying the instrument, the bearing of the points A, E, the termini of the error, is found to be N. 69° 50′ E., precisely the reverse bearing of BC; and the distance exactly 10 chains. It is certain then that an error of 10 chains was made in chaining the distance BC.

Had the point E in plotting fallen at F, it would have shown also a very probable error of 4 chains in chaining AB.

In a similar manner *errors in bearing* may be detected;

whether they arise from a mistake of the surveyor, or from disturbance of the needle by local attraction or other cause.

Such is the readiness with which errors are detected by the Trigonometer, that any marked error is usually even *located* by a bare *glance of the eye.*

After all, even where unusual skill is employed in obtaining the field data, small errors will sometimes occur, for which the surveyor is at a loss to account; uncertain whether to attribute them to chaining or to the needle. In a case of this kind, the general rule is, to assign the error to the *most probable locality* in the survey.

If he still detects no cause of the error, he must presume that it is *where the plotting shows it to be ;* either in distance or angle, or both ; and he must make the correction accordingly. But before he does so, he should be certain, by carefully *reviewing his work,* that he has made no error in *plotting.*

If the error is quite small, a quick, and sufficiently accurate way is, first exactly to bisect by a dot the right line joining the two terminal dots; and then to change in position and length, but not bearing, the terminal sides, and the other sides so far as necessary, so as to make this dot the angular point.

But if the error is considerable, and the cause does not appear, the field work must be performed anew sufficiently far to detect the error.

(47.) TRIANGULATION.

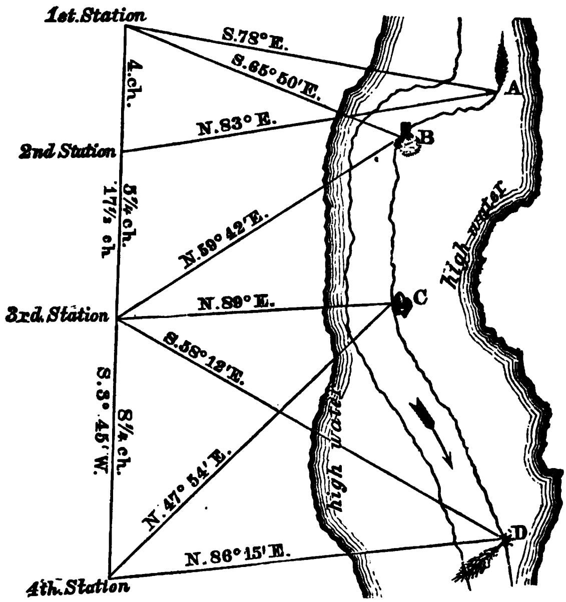
In no department of surveying is the great utility of the Trigonometer more manifest than in obtaining the position and distances of inaccessible objects, the width and course of rivers, the outline of marshes, mill ponds, small lakes, or rocky tracts by *Triangulation.* Since its use in these cases wholly sets aside trigonometrical calculations, it is easy to see that in extended operations *three fourths,* and sometimes even *four fifths of the labor is saved.*

Fig. 19, furnishes an example of this sort, in which it is required to ascertain the bearing of several inaccessible objects

situated at angular points, in order to locate the *east bank* of a river overflowed by high water.

From the data given in the figure find the courses and distances of lines joining AB, BC, and CD.

FIG. 19.



Ans. AB is found to be S. 67° 16′ W. 3.48 chains.

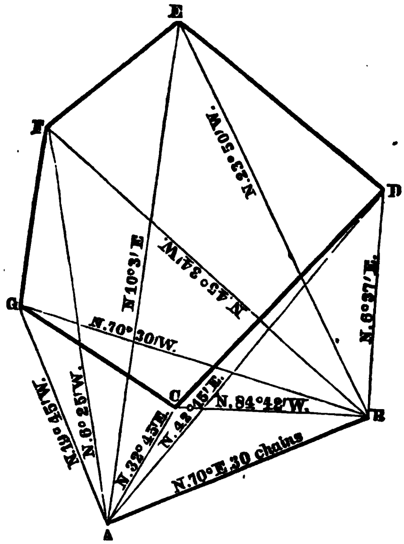
AB is found to be	S. 67° 16′ W.	3.48 chains.
BC	"	"	S. 5° 5′ W.	5¼ "
CD	"	"	S. 23° 6′ E.	8.22 "

(48.) SURVEY FROM TWO STATIONS.

The same method of operation is employed in surveying a field from *two stations*, either within or without the field.

Given Fig. 20, the *length* and *bearing* of the *base line* AB, with the *bearings* of lines drawn from each of the stations A and B, to each of the angles of the field; to find the courses and distances of the sides CD, DE, EF, FG, GC. The answer may be found in the columns of Courses and Distances in example 5. (Art. 60.)

Fig. 20.



It is evident from the two last examples, that where the nature of the ground will allow, such are the advantages of the use of the Trigonometer in common triangulation, that this process is often preferable to the usual one of surveying in detail with chain and compass the several sides of the field.

(49.) ATMOSPHERIC CHANGES.

When in order to make a re-survey, or for any other cause the plot of the survey is *allowed to stand several hours*, especially if in the interval the atmosphere should change in *temperature or humidity*, it should be *tested*, and if necessary, the field *re-plotted* before calculating the area.

It may be remarked in general, that the table ought to stand in a dry room of comfortable and uniform temperature while being used, and for some time previously; otherwise a slight error may occur in the area of the field, arising from the expansion or contraction of the paper.

For the same cause, whenever any *accurate plan or map* is

drawn, the *scale of units* used in its delineation *should always be appended.* Thus, whatever contraction or expansion the changes in the condition of the atmosphere may produce in the *map*, the same contraction or expansion will take place in the *scale.*

FIRST METHOD OF DETERMINING AREAS.

(50.) BY PARALLELOGRAMS.

When the sides of the field whose contents are to be ascertained are *equal* and *parallel*, the simple rule is, *Multiply the base by the perpendicular height.* The product will be the area in *square chains.*

If the parallelogram be *rectangular*, the Trigonometer will not be required; since *the length multiplied by the breadth* will give the area. But let it never be forgotten that when ever the angles are *oblique*, the *perpendicular* not *slant* height must be obtained by measure with the Scale-Plate.

(51.) CORRECTIONS FROM VARYING THE SCALE.

Should it be necessary to *vary the scale*, in order to produce a plot suited to the size of the draughting table (Art. 21); particular care should be taken, after the *area of the plot* is obtained to make the *proper correction* required for deducing the true *area of the field.*

Suppose that the given distances as is usual are *chains*: and that in order to produce a *plot of the right dimensions* they are *multiplied* by $\frac{4}{10}$. In making the correction necessary for obtaining the *area of the field*, the area of the *plot* must be *divided* by the *square* of this fraction; that is $\frac{16}{100}$.

Again, suppose the distances are *multiplied* by $\frac{3}{10} = .3$. Then the area of the plot *divided* by $.3 \times .3 = .09$, will be the area of the field.

If the *multiplier* be $.5 = \frac{1}{2}$, the *divisor* will be $.25 = \frac{1}{4}$. But in obtaining the latter area it is generally found convenient to get first the *double area.* Hence in the case supposed, instead of dividing the area of the plot by $\frac{1}{4}$, that is, multiplying by *four*, to obtain the area of the field; if *twice* the former area be multiplied by *two*, the same result would be obtained.

The simple rule is, *Whatever multiplier or divisor is employed for varying the length of the sides before plotting, the* **square** *of this number or fraction must be applied to the area of the plot, by a* **contrary process,** *in order to obtain the true area of the field.*

SECOND METHOD.

(52.) BY TRAPEZOIDS AND TRIANGLES.

For a **Trapezoid,** the rule is, *Multiply the sum of the two parallel sides by the perpendicular distance between them. Half the product will be the area.*

For a **Triangle,** *Multiply the base by the perpendicular height, and halve the product.*

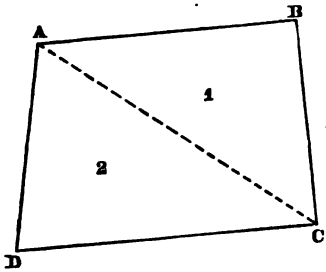
(53.) DIVISION OF THE PLOT.

For the sake of uniformity and the consequent prevention of mistakes, by this method all irregular figures should be divided or supposed to be divided into *triangles* or *trapezoids,* and designated by numeral figures, as in the examples next given.

Example 1. Given the following courses and distances to find the area of the field. Fig. 21.

1.	N. 85° 10′ E.	16.50	chains.
2.	S. 5° 25′ E.	13.02	"
3.	S. 85° 10′ W.	19.12	"
4.	N. 5° 59′ E.	13.25	"

FIG. 21.



When it is required to divide a plot into triangles it should be so divided as to make the sides as nearly *equal* as possible. Hence, in this example it is better to join AC, than BD.

The angle B is so nearly a right angle that BC is very nearly the perpendicular height of the two triangles. The base of triangle 1 is AB; and DC that of triangle 2.

OPERATION.

$$
\begin{array}{lrl}
AB & = 16.50 & \\
DC & = \underline{19.12} & \\
Sum & = 35.62 & \\
BC & = \underline{13.02} & = \text{Perpendicular.} \\
 & 7124 & \\
 & 10686 & \\
 & 3562 & \\
 & 20)\overline{463.7724} & = \text{Double Area in Square Chains.} \\
 & 23.18862 & = \text{Area in Acres.}
\end{array}
$$

Ex. 2. What is the area of the field whose courses and distances are as follows? Fig. 22.

1.	S. 5° 25′ W.	21.05	chains.
2.	N. 89° 10′ E.	19.30	"
3.	S. 5° 25′ W.	7.42	"
4.	S. 69° 20′ E.	3.08	"
5.	N. 27° 38′ E.	33.49	"
6.	S. 89° 10′ W.	35.12	"

After plotting the field in this example, produce BC to G, and join CE.

The area of trapezoid No. 1, $= \dfrac{(AF+BG)\times AH}{2}$.

" " triangle No. 2, $= \dfrac{CE\times GK}{2}$.

" " " No. 3, $= \dfrac{CE\times DL}{2}$. Or the whole

$$\text{Area} = \frac{(AF+BG)\times AH + CE\times GK + CE\times DL}{2}.$$

Or, The double area of trapezoid No. 1,	= 1295.244	sq. ch.
" " " triangle No. 2,	= 64.216	"
" " " " No. 3,	= 22.044	"
Twice the area,	= 1381.504	"
or, Area	= 69.0752	Acres.

FIG. 22.

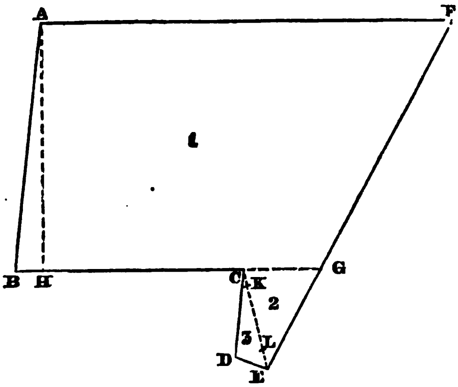
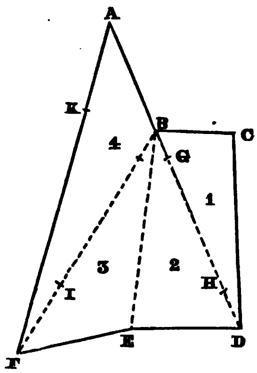


FIG. 23.



Ex. 3. Fig. 23.

1.	S. 25° 0′ E.	6.25 chains.
2.	East,	4.10 "
3.	S. 4° 8′ E.	10.41 "
4.	S. 87° 40′ W.	5.73 "
5.	S. 75° 55′ W.	6.32 "
6.	N. 13° 45′ E.	18.34 "

Required the area.

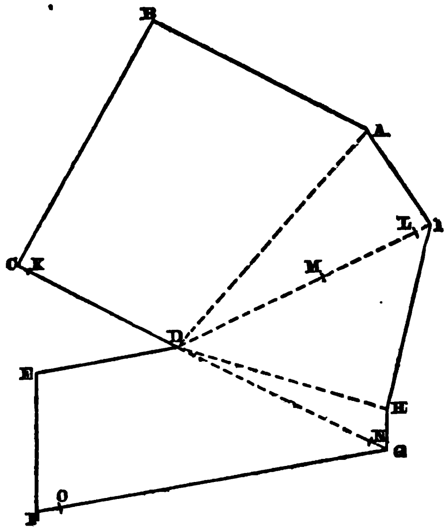
Ans. Triangle 1,	= 42.56	sq. ch.	÷ 2.
" 2,	= 60.	"	÷ 2.
" 3,	= 84.32	"	÷ 2.
" 4,	= 112.40	"	÷ 2.
Double Area	= 299.28	Square Chains.	
or, Area	= 14.964	Acres.	

Ex. 4. Fig. 24. Given the following courses and distances, to find the area of the field.

1.	N. 62° 45′ W.	25.20 chains.
2.	S. 29° 25′ W.	29.35 "
3.	S. 62° 45′ E.	18.95 "

4.	S. 79° 56′ W.	15.04 chains.
5.	South,	14.65 "
6.	N. 79° 56′ E.	37.30 "
7.	N. 0° 18′ E.	4.16 "
8.	N. 13° 35′ E.	20.05 "
9.	N. 34° 11′ W.	11.87 "
		Area = 147.46 Acres.

FIG. 24.



Some authors multiply the methods of finding the content of land to such an extent as to bewilder rather than aid the student; many of them serving more for gratification to the curious, than for utility to the practical surveyor. To those who use the *Trigonometer*, the methods given in this section embrace all that will be of value in ordinary practice.

The following is the most reliable and convenient when the field has many sides.

THIRD METHOD.

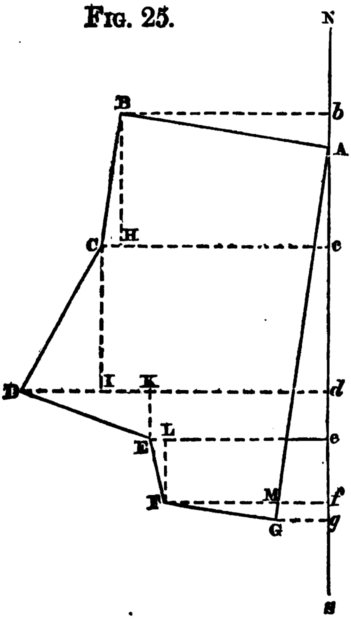
(54.) BY LATITUDE AND LONGITUDE.

This method may be illustrated by an example.

Given the following courses and distances to find the area of the field.

1.	N.	80° 40′	W.	9.00	chains.
2.	S.	8° 0′	W.	5.68	"
3.	S.	28° 55′	W.	7.00	"
4.	S.	70° 20′	E.	5.83	"
5.	S.	12° 30′	E.	2.75	"
6.	S.	81° 20′	E.	4.75	"
7.	N.	8° 13′	E.	15.82	"

FIG. 25.



Let ABCDEFG, Fig. 25, be the perimeter of the field. Through A, draw the meridian NS. From the several angular points, B, C, D, E, F, G, draw to the meridian the parallels of latitudes B*b*, C*c*, D*d*, E*e*, F*f*, G*g*. These will divide the figure *b*BCDEFG*g*, into the trapezoids *b*BC*c*, *c*CD*d*, *d*DE*e*, *e*EF*f*, *f*FG*g*; and form the triangles AB*b*, AG*g*. The area of the field is manifestly equal to the difference between the sums of the trapezoids and of the triangles.

Hence, $(Bb+Cc)\times BH+(Cc+Dd)\times CI+(Dd+Ee)\times EK+(Ee+Ff)\times FL+(Ff+Gg)\times GM-(Bb\times Ab+Gg\times Ag)=$ twice the area of the field.

It will greatly add to convenience in the operation, and tend

to prevent mistakes, to arrange in a table as follows, the **No. of the several Sides, Courses, Distances, Northings, Southings, Longitude, Double Longitude, North and South Areas.**

No.	Courses.	Chains.	N.	S.	Lon. 0.	D. Lon.	N. Areas.	S. Areas.
1	N. 80° 40′ W.	9.00	A*b*		B*b*	0+B*b*	B*b*×A*b*	
2	S. 8° 0′ W.	5.68		BH	C*c*	B*b*+C*c*		(B*b*+C*c*)×BH
3	S. 28° 55′ W.	7.00		CI	D*d*	C*c*+D*d*		(C*c*+D*d*)×CI
4	S. 70° 20′ E.	5.83		EK	E*e*	D*d*+E*e*		(D*d*+E*e*)×EK
5	S. 12° 30′ E.	2.75		FL	F*f*	E*e*+F*f*		(E*e*+F*f*)×FL
6	S. 81° 20′ E.	4.75		GM	G*g*	F*f*+G*g*		(F*f*+G*g*)×GM
7	N. 8° 13′ E.	15.82	*g*A		0	G*g*+0	G*g*×*g*A	

TABLE WITH THE FIGURES.

No.	Courses.	Chains.	N.	S.	Lon. 0.00	D. Lon.	N. Areas.	S. Areas.
1	N. 80° 40′ W.	9.00	1.46		8.88	8.88	12.9648	
2	S. 8° 0′ W.	5.68		5.62	9.67	18.55		104.2510
3	S. 28° 55′ W.	7.00		6.13	13.05	22.72		139.2736
4	S. 70° 20′ E.	5.83		1.97	7.56	20.61		40.6017
5	S. 12° 30′ E.	2.75		2.68	6.96	14.52		38.9136
6	S. 81° 20′ E.	4.75		.72	2.26	9.22		6.6384
7	N. 8° 13′ E.	15.82	15.66		0.00	2.26	35.3916	
							48.3564	329.6783
								48.3564
								20)281.3219
							Number of Acres	14.06609.

(55.) MERIDIANS.

A meridian may be drawn through the extreme eastern angle of the plot, as in the example just given, or through the extreme western; but it is not essential that a meridian should be drawn at all. The East and West edges of the Draughting Table constitute *permanent meridians.*

If the Scale-Plate be so placed upon the metallic Border of the table that the lips on one side of its Guides shall come in contact with the east or west edge of the table; the lips on the other side of the Guides will rest upon the paper about half an inch from the inside of the Border. Now by sliding the Scale-Plate along the Border, keeping the *outer lips* in contact with its edge, the *Grooves* of the *inner lips* will traverse a straight line exactly parallel with the edge of the

table, which may be called the *Latitude Meridian*; that is, *a convenient line on which to measure the difference of latitude of the angular points of the plot,* but merely *imaginary.*

Again, if the zero of the Protractor Vernier be set at N. on the limb (Art. 3), its base or lip brought in contact with the *same edge* of the table as before, and the Scale-Plate be laid upon the Rule with the *Attached Guide* by its knob held firmly in contact with the *base* of the instrument, and the latter in close contact with the *table edge,* each of the several *Grooves* of the former will lie in what may be called the *Zero Meridian* of its corresponding scale. And by sliding the base of the instrument along the table edge, each of these Grooves will traverse its own meridian.

This is the normal, uniform position of the instrument for *measuring the longitude* of the several angular points of the plot. By sliding the instrument in latitude, and the Vernier Guide in longitude, till the *unit groove* (Art. 9), of the latter exactly coincides with *any angular point, its longitude* is directly indicated by the scale and vernier selected for the given case.

Let the learner never forget to keep the *base* of the Trigonometer and the *Attached Guide closely* in their *normal position* at the moment of *measuring* the *longitude.* (Art. 56, 3.)

(**56.**) From the foregoing illustrations we derive the following

RULE.

1. Across the *Latitude Meridian* from each angular point draw *parallels of latitude*; or instead *short parallels intersecting* this meridian.

2. Place in the column of Northings or Southings, opposite each course, and obtained with the Scale-Plate from the Latitude Meridian, its own *Northing* or *Southing.*

3. In the *Longitude column,* opposite the first course, place the *corresponding Longitude,* measured from the latter terminus of the course; and proceed in the same manner with all the remaining sides; placing the longitude of the last

course, whether it be 0, or a meridian distance, both opposite that course and at the head of the longitude column.*

4. *Add together* in the Longitude column the head number, and the number opposite the first course, and place the sum in the *Double Longitude column* opposite the same course. Place the sum of the first and second numbers opposite the second course; the sum of the second and third opposite the third course, &c., through all the remaining sides.

5. *Multiply the Double Longitude* belonging to each course by its *corresponding Northing* or *Southing*; placing the product in the column of areas of the *same name*, and opposite that course. The difference between the *sum of the North Areas*, and the *sum of the South Areas* will be equal to *twice the area of the Plot, in square units* of the *same name* as the *given distances*. (Art. 54.) If the distances are *chains*, that difference will be *twice the area of the field.*

(57.) TEST OF WORK.

After the Longitudes, and the Northings and Southings of the several courses are obtained, they should be *tested* by re-measuring the former, and comparing the sums of the two latter, which should be equal. Indeed the entire operation of determining areas, like that of obtaining the field data, and plotting, requires great care. A mistake of one figure in the column of Double Longitudes, for instance, may occasion the owner of the field surveyed or some one else, the loss of hundreds of dollars, or a most perplexing and expensive lawsuit.

The use of the *Trigonometer* however, so greatly simplifies the methods of determining areas, and correcting errors, that with it the Surveyor need never allow an error to go out of his hands.

(58.) ADDITIONAL PLATE.

It is sometimes desirable in obtaining the Longitude, to place upon the Rule between the Attached Guide and the base

* If the longitude is *less than a unit*, the *first* method of measuring (Art. 10), must be used.

of the Trigonometer a short *metallic plate* of exactly known length and breadth, in order either to throw the zero meridian farther from the edge of the table than it would otherwise fall, or to increase the available length of the Scale-Plate. But it should always be recollected that whenever used for the latter purpose, the distance employed, whether the length or breadth of the plate, must be *added to the Longitude.*

(59.) PLOTTING BY DOTS.

When the surveyor or student has made himself familiar with the method of determining areas by Latitude and Longitude, he will find it more expeditious to plot the field according to directions given (Arts. 25 and 42); viz.

First, plot the field by merely *dotting with the prick the several angular points:* enclosing in a little circle, with a common soft pencil, each dot when made, so that it may be readily seen, *without drawing the sides at all, or even a meridian;* the points with their corresponding intersecting parallels being designated by the numerals 1, 2, 3, 4, &c. The

Fig. 26.

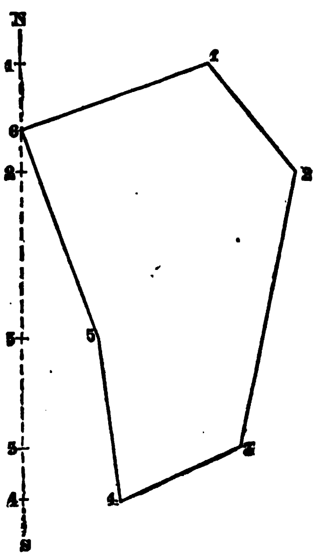
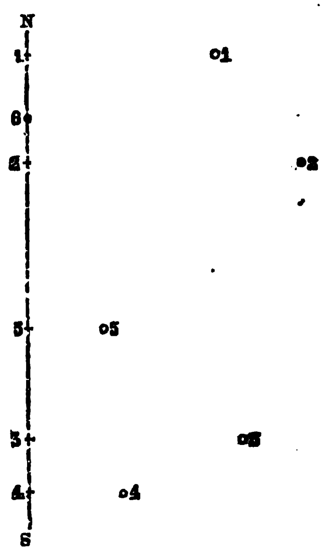


Fig. 27.



Zero and Latitude meridians may, just as well, indeed better be *supposed* drawn than to be *actually* drawn.

These directions may be illustrated by the following example :

Ex. 4. Figs. 26, 27. Given the following courses and distances to find the area of the field. Fig. 26, gives the outline, Fig. 27, merely the angular points. The *Zero* and *Latitude meridians* in this example are identical.

No.	Courses.	Chains.	N.	S.	Lon. 0.00	D. Lon.	N. Areas.	S. Areas.
1	N. 71° 10′ E.	12.50	4.03		11.83	11.83	47.6749	
2	S. 38° 35′ E.	8.75		6.84	17.29	29.12		199.1808
3	S. 11° 55′ W.	17.85		17.46	13.60	30.89		539.3394
4	S. 66° 40′ W.	8.30		3.29	5.98	19.58		64.4182
5	N. 6° 45′ W.	10.35	10.28		4.76	10.74	110.4072	
6	N. 19° 43′ W.	14.11	13.28		0.00	4.76	63.2128	
							221.2949	802.9384
								221.2949
							20)	581.6435
							Number of Acres.	29.08217.

(**60.**) In the following example the Zero Meridian is supposed drawn $1\frac{1}{4}$ chain east of the extreme eastern angular point of the field. The longitude of that point is therefore 1.25 chain; which by (Art. 56, 3), must be placed both opposite the last course, and at the head of the Longitude column. The courses and distances may be obtained from the data given in Fig. 20, (Art. 48.)

EXAMPLE 5.

No.	Courses.	Chains.	N.	S.	Lon. 1.25.	D. Lon.	N. Areas.	S. Areas.
1	N. 48° 59′ W.	28.61	18.77		22.85	24.10	452.3570	
2	S. 54° 3′ W.	17.49		10.27	37.01	59.86		614.7622
3	S. 10° 20′ W.	19.72		19.41	40.55	77.56		1505.4396
4	S. 55° 38′ E.	19.56		11.05	24.41	64.96		717.8080
5	N. 46° 32′ E.	31.93	21.96		1.25	25.66	563.4936	
			40.73	40.73			1015.8506	2838.0098
								1015.8506
							20)	1822.1592
							Number of Acres.	91.10796

SHIFT OF BOARD.

(61.) If the *length* of a field is much greater than the *breadth*, and the sides corresponding with the former are within a few degrees of coincidence with a parallel of latitude passing through it, it is sometimes convenient in obtaining the data for the columns of Northings, Southings, and Longitude, to consider the sides of the Draughting Board *shifted one fourth of a revolution to the right or left.* The north or south sides then become the east or west sides.

The following example will illustrate the operation.

1.	N. 78° 0′ W.	9.96	chains.
2.	N. 27° 35′ W.	2.20	"
3.	West,	17.22	"
4.	S. 1° 50′ E.	5.96	"
5.	East,	17.50	"
6.	South,	4.88	"
7.	S. 84° 40′ E.	9.32	"
8.	N. 7° 42′ E.	7.82	"

Let the board be supposed to revolve 90° from left to right. *East* then becomes *North*, *West—South*, *South—East*, and *North—West*; as in the subjoined table.

No.	Courses.	Chains.	N.	S.	Lon. 10.05	D. Lon.	N. Areas.	S. Areas.
1	N. 78° 30′ W.	9.96		9.76	8.05	18.10		176.656
2	N. 27° 35′ W.	2.20		1.02	6.10	14.15		14.433
3	West.	17.22		17.22	6.10	12.20		210.084
4	S. 1° 50′ E.	5.96	.18		12.04	18.14	3.2652	
5	East.	17.50	17.50		12.04	24.08	421.4000	
6	South.	4.88	0.00		16.92	28.96		
7	S. 84° 40′ E.	9.32	9.28		17.79	34.71	322.1088	
8	N. 7° 42′ E.	7.82	1.04		10.05	27.84	28.9536	
			28.00	28.00			775.7276	401.173
							401.173	
						20)	374.5546	
							18.72773	Acres.

The advantage of this change is, that the longitude of the farther angular points being thus greatly diminished, that of each can be obtained by once setting the Vernier Guide, instead of twice.

SECTION IV.

DIVIDING AND LAYING OUT LAND.

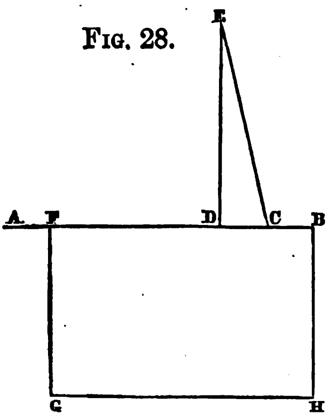
When it is required to lay out a given area in a *rectangular form,* the process being arithmetical, the Trigonometer is unnecessary, except in the following case.

PROBLEM I.

(**62.**) *To lay out a given area in a rectangular form, having the* **length to exceed the breadth by a given difference.**

With the instrument set at its meridian draw a straight line AB, of indefinite length; from an assumed point C in this line, lay off both ways half the given difference; from D the end of one of these, at right angles draw a line equal to the square root of the given area; join its latter terminus E, to the assumed point C. The half difference CB or CD added to CE, equals the length, and subtracted therefrom equals the breadth of the required parallelogram. With the data thus obtained the figure may be readily completed.

FIG. 28.



The following example illustrates the operation. Fig. 28.

It is required to lay out 48.8 acres in the form of a parallelogram, whose length is to exceed its breadth by 10 chains.

With the Trigonometer set at its meridian draw indefinitely the straight line AB; from an assumed point C in this line make CB and CD each equal to 5, half the given difference; from D, at right angles to CD draw DE, equal to 22.09 chains, the square root of 488 square chains,

the required area. Then CE, = 22.65. Make CF = CE; therefore BF = CF + CB = 22.65 + 5 = 27.65 chains, is the length; and CF − CB = DF = 17.65 chains, the breadth of the required parallelogram. Lastly, with these data complete the figure if desired.

PROBLEM II.

(**63.**) *To lay out a given area in the form of a* **Triangle or Parallelogram** *one side and an adjacent angle being given.*

FOR A TRIANGLE.

Divide twice the given area in **square** chains, by the given side in **linear** chains.

FOR A PARALLELOGRAM.

Divide the given area by the given side. The quotient in both cases will be the **perpendicular height** of the required figure, which after the given angle is laid down must be measured off with the Scale-Plate at right angles to the given side. The figure may then be completed, and the other parts measured if necessary.

Fig. 29.

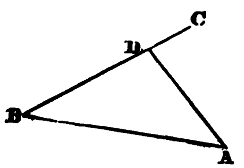
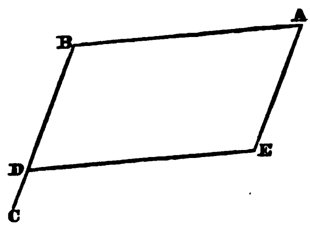


Fig. 30.



Ex. 1. Given AB, Fig. 29, N. 82° 15′ W. 22.05 chains, and BC, N. 61° 5′ E, to lay off a triangle containing 10 acres, by a line drawn from A to BC. Required the length and bearing of the unknown sides AD, and BD.

Ans. AD, N. 39° 40′ W. 13.4 chains; BD, 15.19 chains.

Ex. 2. Given AB, Fig. 30, N. 5° 10′ W. 47.85 chains, and BC, N. 68° 45′ W., to lay off a parallelogram containing 120

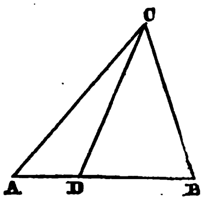
acres, by lines running parallel to AB and BC. Required the length of the side BD. *Ans.* 28.09 chains.

PROBLEM III.

(64.) *The area and base of a triangle being given, to cut off a given part of the area* **by a line running to the base from the opposite angle.**

Triangles of the same altitude are to one another as their bases. Hence, Fig. 31.

FIG. 31.



ABC : ADC : : AB : AD.

or,

The area of the given triangle
Is to the area of the part to be cut off;
As the base of the former
To the base of the latter.

In this problem the Trigonometer will be required only for *plotting* the figure, and *measuring the sides and angles* of the two triangles.

Ex. It is required to lay off from a triangular field of 15 acres, one of whose sides is 75 rods long, $5\frac{1}{2}$ acres, by a line running to it from the opposite angle. What is the length of the base of the smaller triangle?

Ans. 27.5 rods.

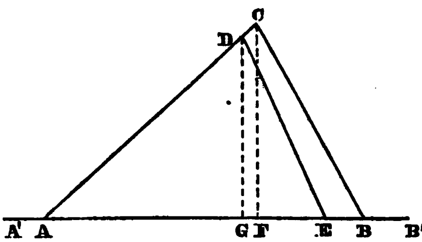
PROBLEM IV.

(65.) *The area and two sides of a triangle being given, to cut off a triangle containing a given area,* **by a line running from a given point in one of the given sides, and falling on the other.**

If the angle included by the given sides be given; with their given bearings draw those sides, of their given length or indefinitely. Then having found the given point by measure, divide twice the area to be cut off, in square chains, by the perpendicular height in linear chains of the given point above the opposite line or base. The quotient will be the base of the required triangle.

If the included angle be not given, the perpendicular height of the triangle may be found by dividing twice the given area by the base. The triangle may then be drawn substantially according to Case IV, in Oblique Angled Triangles (Art. 40), as shown in the following example.

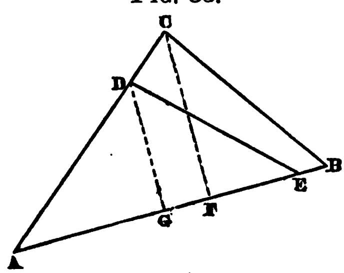
Fig. 32.



Ex. 1. Given the area of the triangle ABC, Fig. 32, 8.45 acres; AB, 16.77 chains; AC, 15.05 chains; AD, 14.03 chains; and the area of the triangle ADE, 6.975 acres, to find the base AE, of the required triangle.

As the angle A is not given, divide twice the area by the base. Hence $\frac{169}{16.77} =$ CF. Draw CF as a meridian. Through F draw both ways a parallel of latitude A′B′ of indefinite length. From C draw the given side CA, = 15.05 chains, meeting A′B′ in the point A. From A measure off the given distances AB = 16.77 chains, and AD = 14.03 chains. Then according to the rule, $\frac{139.5}{DG} =$ AE, = 14.75 chains, the required side.

Fig. 33.



Ex. 2. Given AB, Fig. 33, N. 75° 10′ E., 54.80 chains; AC, N. 34° 50′ E., 45 chains; AD, 34.6 chains; and the area of the triangle ABC = 80 acres, to cut off the triangle ADE = 56 acres. Required the length of the base AE. *Ans.* 50.02 chains.

PROBLEM V.

(66.) *The* **bearings** *of* **two adjacent sides** *of a tract of land being given, to cut off a* **Triangle** *containing a given area,* **by a line running a given course.**

With the bearing of the line to be run, draw a line **one chain**, or **any convenient unit** in length and call it the **unit line.** Through one end of this line, with one of the given bearings, draw a line of indefinite length; and through the other end, with the other bearing draw a line meeting the indefinite line. From the angular point draw a line *perpendicular to the unit line*, and call its length the **divisor.**

With the divisor thus obtained divide twice the required area in square chains, and extract the square root of the *quotient.* The result will be the *length of the required side.* Its *perpendicular distance* from the angular point may be obtained by dividing by it twice the given area.

DEMONSTRATION.

Let CEF, Fig. 34 or 35, = the given area, $= A$; and $AB = 1$. Then $ABC : A :: AB^2 : EF^2$.

$$\text{But (Art. 52,) } ABC = \tfrac{1}{2}\,\frac{CD}{AB} = \tfrac{1}{2}\,CD.$$

$$\therefore\ \tfrac{1}{2}CD : A :: 1 : EF^2;$$

$$\therefore\ \tfrac{1}{2}CD \times EF^2 = A;$$

$$\therefore\qquad EF^2 = \frac{A}{\tfrac{1}{2}CD} = \frac{2A}{CD};$$

$$\therefore\qquad EF = \sqrt{\frac{2A}{CD}}.$$

$$\text{And}\qquad CG = \frac{2A}{EF}.$$

FIG. 34.

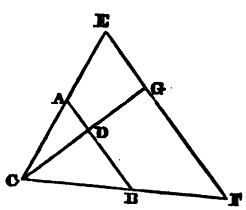
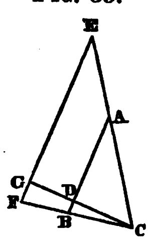


FIG. 35.



It is not essential *what distance* is taken as the **unit line.** *But in all cases the* **divisor** $= \frac{CD}{AB}$. This remark applies to both

the 5th and 6th Problems. It will be well to use *a unit of as great length* as the Draughting Board will allow. This will secure greater accuracy than a *short* unit line.

Ex. 1. The bearing CE, Fig. 34, of one side of a field is N. 26° 54′ E.; the bearing of the adjacent side CF, S. 87° 15′ E.; the area to be cut off, $9\frac{1}{4}$ acres; and the bearing of the cutting line, N. 39° 5′ W. What is the length of the cutting line EF, and its perpendicular distance from the angular point C?

Ans. EF = 16.17 chains; and CG = 12.06 chains.

Ex. 2. Given the bearing of a side of a tract of land CE, Fig. 35, N. 10° 18′ W.; the bearing of the adjacent side CF, N. 75° 55′ W.; the area to be cut off, 45.3 acres; and the bearing of the cutting line EF, S. 24° 38′ W. Required the length of each of the three sides of the triangle.

Ans. EF = 38.29 ch.; CE = 41.32 ch.; and CF = 24.08 ch.

PROBLEM VI.

(67.) *The* **bearings** *of* **three adjacent sides** *of a tract of land, and the* **length** *of the* **middle sides** *being given; to to cut off a* **Trapezoid** *containing a given area, by a line* **parallel to the given side.**

With the bearing of the side whose distance is given, draw a **unit line;** with this line and the bearings of the two other sides, draw a triangle, and *obtain a* **divisor** *and* **quotient,** precisely *as in the last problem.*

If the line to be run be *longer* than the given side, ADD to the square of this side the *quotient* thus obtained; but if the line to be run be *shorter* than the given side, SUBTRACT *the quotient.* The *square root of the sum or difference will be the length of the required line.*

Fig. 36.

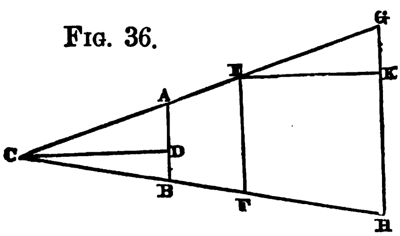
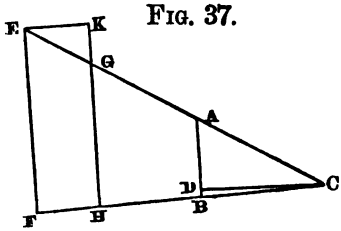


Fig. 37.



The perpendicular between the two parallel sides may be obtained by dividing *twice the required area* in square chains by the *sum of those sides.*

DEMONSTRATION.

Let GE, EF, FH, Figs. 36, 37, be the three adjacent sides of a tract of land whose bearings are given, and EF the given side.

Produce the sides GE, and HF, till they meet in the point C. Draw AB parallel to EF, and at such perpendicular distance from the point C that AB shall equal a unit. Draw CD perpendicular to AB, and EK perpendicular to GH, or GH produced. Let FEGH be the trapezoid containing the given area to be cut off; of which the side GH is parallel to EF.

Let $FEGH = A$, and $AB = 1$.

Then, as in the last problem, $ABC = \frac{1}{2}CD$.

$\therefore \quad \frac{1}{2}CD : CEF :: 1 : EF^2$;

$\therefore \quad CEF = \frac{1}{2}CD \times EF^2$.

Again, $\frac{1}{2}CD : CGH :: 1 : GH^2$;

but $CGH = CEF \pm A$;

$\therefore \quad \frac{1}{2}CD : CEF \pm A :: 1 : GH^2$;

or $\frac{1}{2}CD : \frac{1}{2}CD \times EF^2 \pm A :: 1 : GH^2$;

$$\therefore \quad 1 : EF^2 \pm \frac{A}{\frac{1}{2}CD} :: 1 : GH^2;$$

$$\therefore \quad GH^2 = EF^2 \pm \frac{A}{\frac{1}{2}CD} = EF^2 \pm \frac{2A}{CD};$$

$$\therefore \quad GH = \sqrt{EF^2 \pm \frac{2A}{CD}}.$$

$$\text{And } EK = \frac{2A}{EF + GH}.$$

After plotting FEGH, the trapezoid to be cut off, the length of the sides EG and FH may be obtained with the Scale-Plate.

Ex. 1. Given the bearings of the three adjacent sides of a tract of land, GE, EF, FH, Fig. 36, and the length of the side EF, as follows: GE, S. 69° 50′ W.; EF, S. 2° 18′ E.,

9.85; FH, S. 81° 15′ E. It is required to cut off by a line parallel to EF, 15 acres. What is the length of the required line GH, and the perpendicular EK?

With the bearing S. 2° 18′ E., draw AB as a **unit line**; from B, with the bearing S. 81° 15′ E., draw BC of indefinite length; and from A, with the bearing S. 69° 50′ W., draw AC meeting BC in the point C. The perpendicular CD, obtained as in the last problem is the **divisor.** With this distance divide twice the area to be cut off; and to the square of 9.85 chains, the given side, *add the quotient.* The square root of the sum will be the *length of the required side* GH. And 300 chains, twice the given area, divided by the sum of the sides EF and GH, will give the *perpendicular* EK.

Ans. GH = 15.88 chains; and EK = 11.66 chains.

Ex. 2. Given Fig. 37, the bearing of the side HF of a field, N. 5° 2′ W.; FE, N. 86° 55′ E., 15½ chains; EG, S. 27° 30′ W. It is required to cut off by a line GH parallel to EF, 7½ acres of land. What is the length of GH, and of the perpendicular EK?

Ans. GH = 12.10 chains; and EK = 5.44 chains.

PROBLEM VII.

(68.) *The bearings of three adjacent sides of a tract of land, and the length of the middle side being given, to cut off a* **Trapezium** *containing a given area, by a line running a* **given course.**

With the bearing and distance of the given side, the bearing of the line to be run, and of one of the adjacent sides, **draw a Triangle** forming **a part of the required Trapezium.** Find its area, and **subtract it** from the **given area.**

Then calling that side of the triangle which is parallel with the line to be run, the **middle side,** cut off according to problem VI., **a Trapezoid containing the remaining area.**

Ex. 1. Given the bearings of the three adjacent sides CA, AB, BE, Fig. 38, with the length of the middle side as follows, viz.: CA, N. 75° 10′ W.; AB, N. 12° 45′ E., 18.35 chains;

BE, N. 89° 56′ E., to cut off the trapezium GABF, containing $32\frac{1}{2}$ acres, by a line GF, running N. 1° 25′ W. What is the length of each of the three sides GF, FB, and AG?

With its given bearing and distance draw AB; from the point A (whose perpendicular distance from the line to be run must always be *greater than that from B*), with its given bearing draw AC; and from B, with the bearing of the line to be run draw BD. And having found the area of the triangle ABD, (Arts. 52, 53), subtract it from 325 chains, the given area. Then calling BD the *middle side*, cut off, according to problem VI, the trapezoid BDGF.

Ans. GF = 22.71 ch.; FB = 13.49 ch.; and AG = 18.29 ch.

FIG. 38.

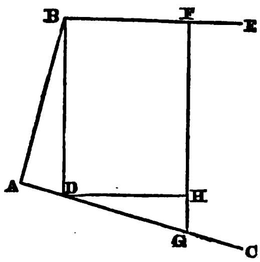
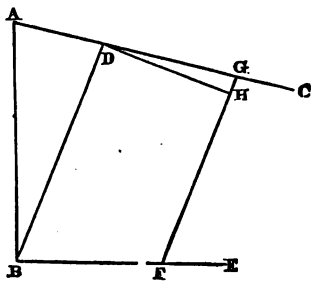


FIG. 39.



Ex. 2. Given the bearings of the three adjacent sides of a tract of land CA, AB, and BE, Fig. 39, with the length of the middle side, as follows, viz.: CA, N. 75° 20′ W.; AB, South, $15\frac{1}{4}$ chains; BE, S. 88° 35′ E.; to cut off the trapezium GABF, containing $19\frac{1}{2}$ acres, by a line GF, running S. 22° 48′ W. Required the length of the three sides AG, GF, and FB.

Ans. AG = 13.73 ch.; GF = 12.99 ch.; and FB = 8.25 ch.

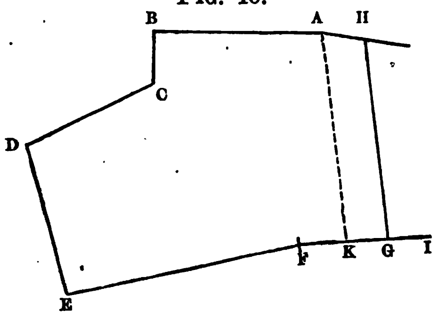
PROBLEM VIII.

(69.) *The bearings of several adjacent sides of a tract of land, and the distance of each* **except the first and last** *being given; to cut off* **a given area,** *by a line running a* **given course.**

Plot the field according to (Arts. 23, 43); with the bearing

of the line to be run, join the first angular point to the last, unlimited side, or vice versa; and determine the enclosed area by "Triangles," or "Latitude and Longitude." (Arts. 52–56.) From the given area *subtract the area thus obtained.* Then according to Problem VI, cut off a *trapezoid equal to the remaining area.*

Fig. 40.



Ex. 1. Given the following courses and distances of a tract of land, Fig. 40, to cut off $27\frac{1}{2}$ acres by the line GH, whose bearing is N. 4° 5′ W.

1.	N.	76° 50′ W.	——	chains.
2.	S.	89° 48′ W.	10.85	"
3.	S.	0° 12′ E.	3.25	"
4.	S.	66° 55′ W	8.80	"
5.	S.	11° 8′ E.	10.00	"
6.	N.	78° 52′ E.	15.18	"
7.	N.	86° 25′ E.	——	"

What is the length of the required side GH, and the distance of its termini, G and H, from the angular points, F and A?

With the bearing N. 4° 5′ W., first from the point A, draw to the unlimited side FI, the line AK; and determine the enclosed area. Then by Problem VI, cut off the *trapezoid* AHGK, *equal to the remaining area.*

Ans. AH = 2.71 ch.; HG = 12.67 ch.; and GF = 5.66. ch.

Ex. 2. Given the following courses and distances of a field; to cut off 30 acres by a line running N. 32° 15′ E. meeting the first and last sides.

1.	N.	48° 30′ W.	——	chains.
2.	S.	78° 0′ W.	8.00	"
3.	N.	26° 30′ W.	11.08	"
4.	N.	38° 30′ E.	12.82	"
5.	S.	64° 0′ E.	10.86	"
6.	S.	86° 0′ E.	——	"

What is the length of the required side? And what are the distances of its northern and southern termini from the adjacent angular points?

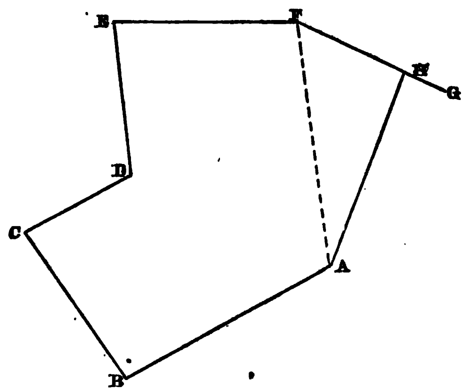
Ans.	Required side	≐ 18.13 chains.				
	Northern terminus	7.34 ch. from angular point.				
	Southern	"	3.50	"	"	"

PROBLEM IX.

(**70.**) *The bearings of several adjacent sides of a tract of land, and the distance of each side except the last being given, to cut off a given area by a line running from the* **starting point to the last side.**

Plot the field, join the first terminus to the last angular point, and determine according to (Arts. 52–56), the enclosed area. Having subtracted this result from the given area, *lay out the remaining area in the form of a* **triangle** according to Problem II.

FIG 41.



Ex. 1. Given	AB, Fig. 41,	S. 61° 5′ W.	14.90	chains.
	BC.	N. 35° 12′ W.	11.18	"
	CD.	N. 61° 5′ E.	7.64	"
	DE.	N. 7° 10′ W.	9.85	"
	EF.	East,	11.87	"
	FG.	S. 65° 25′ E.	——	"

to lay off 32.4 acres by a line AH, running from the point A,

and falling on FG. What is the bearing and distance of AH, and the distance of FH ?

Ans. AH is N. 20° 6′ E., 13.15 ch., and FH = 7.34 ch.

Ex. 2. Given the following courses and distances of a tract of land to cut off $139\frac{1}{2}$ acres by a line running from the starting point and falling upon the last side.

1.	S. 88° 45′ E.	29.30	chains.
2.	N. 48° 18′ E.	9.54	"
3.	N. 2° 25′ E.	42.90	"
4.	S. 48° 18′ W.	28.25	"
5.	N. 48° 42′ W.	18.00	"
6.	N. 83° 50′ W.	——	"

It is required to find the bearing and length of the line to be run, and the distance of its latter terminus from the last angular point.

Ans. Required line, N. 0° 25′ E., 42.02 chains.
Dist. of latter terminus from starting point, 3.32 chains.

PROBLEM X.

(**71.**) *To divide an irregular piece of land into any two given parts.*

Having plotted the field, divide the plot by drawing a line as near as possible to the true division line ; and *determine the area of one of the parts.* If this be *too large or too small,* **add** or **subtract** by the preceding problems a **triangle, trapezoid** or **trapezium,** as the case may require.

For additional examples under the problems of this section the student is referred to the *Treatise on Surveying and Trigonometry,* by STODDARD & HENKLE, before alluded to.